6.2: Geometric Sequences & compound interest.

review: Arithmetic sequences.

ex: 3, 5, 7, 9, ..., 201

$a_n = 3 + 2(n-1)$

$\boxed{a_n = a_1 + d(n-1)}$ ← formula for the nth term of the arithmetic sequence.

$3 + 5 + 7 + 9 + \ldots + 201$ ← 100 terms.

$$S_{100} = \frac{100(3+201)}{2}$$

$$\boxed{S_n = \frac{n(a_1 + a_n)}{2}}$$ ← formula. Sum of n term of an arith. seq.

Geometric sequences.

ex: Find the next few terms

(a) 3, 6, 12, 24, 48, 96, 192, ...

$*2$ $\quad$ $*2$

(b) 54, 18, 6, 2, $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$, ...

$*\frac{1}{3}$ $\quad$ $*\frac{1}{3}$

common ratio of 2 or $\frac{1}{3}$

Formula for the nth term in a geometric seq is:

$\boxed{a_n = a_1 \cdot r^{n-1}}$ where a_1 = 1st term and r = common ratio.

ex: Find a formula for the nth term of:

(a) 7, 21, 63, ...,

$a_n = 7 \cdot 3^{n-1}$.

(b) 192, 96, 48, 24, ...

$a_n = 192\left(\frac{1}{2}\right)^{n-1}$

ex: Find the nth term of the geo. seq if the 3rd term is 24 and the 9th term is 1536.

set up a system

$$\begin{cases} a_3 = 24 = a_1 \cdot r^2 \\ a_9 = 1536 = a_1 \cdot r^8 \end{cases}$$

use substitution

(1) solve for a_1: $a_1 = \frac{24}{r^2}$

(2) sub into 2nd eqt.

$$1536 = \frac{24}{r^2} \cdot r^8$$

$$\Rightarrow \frac{1536}{24} = \frac{24}{24} r^6$$

$$\Rightarrow 64 = r^6$$

$$\Rightarrow r = \sqrt[6]{64} = 64^{1/6} = 2$$

(3) substitute back to find a_1

$$a_1 = \frac{24}{2^2} = 6$$

(4) n^{th} term: $a_n = 6 \cdot 2^{n-1}$

The sum of a geometric seq.

recall:
$$\begin{array}{l} 1+2+3+\dots+100 \\ 100+99+98+\dots+1 \\ \hline 101+\dots\quad+101 \end{array} = \frac{100(101)}{2}$$

ex: Find $6+12+24+\dots+768+1536$

$1536+768+\dots+6$

epic fail: need a new trick.

$$\text{sum} = (6+12+\dots+768+1536)\left(\frac{2-1}{2-1}\right)$$

$$= \frac{\cancel{12}+\cancel{24}+\dots+\cancel{1536}+3072-6-\cancel{12}-\dots-\cancel{768}-\cancel{1536}}{2-1}$$

$$= \frac{3072 - 6}{2 - 1}$$

$$= \frac{6(512 - 1)}{2 - 1}$$

$$= \frac{6(2^9 - 1)}{2 - 1}$$

The sum of n terms of a geo seq. is

$$\boxed{S_n = \frac{a_1(r^n - 1)}{r - 1}}$$

compound interest.

How much will we have in 5 yrs if we save \$1000 today @ 12% interest.

Simple interest

t	\$
0	1000
1	1120
2	1240
3	1360
4	1480
5	1600

Annual compound interest

t	\$
0	1000
1	1120 = 1000 (1.12
2	1254.4 = 1120 (1.12
3	1404.93 = 1254.4 (1.1
4	1573.5 = 1000 (1.12)
5	[illegible]

$$FV = P(1 + rt)$$

$$FV = P(1 + r)^t$$

OR

$$FV = P\left(1 + \frac{r}{n}\right)^{nt}$$

6.2 cont.

Formulas for periodic compound interest.

(1) $FV = P(1+r)^t$

annual compounding.

(2) $FV = P\left(1+\frac{r}{n}\right)^{nt}$

compounding n times/year.

FV = future value

P = present value.

r = interest rate.

n = number of compoundings per year.

t = Number of years.

ex: Find the future value if \$3200 is invested for 7 yrs w/ quarterly compounding @ 9%.

*$FV =$ $\quad r = 0.09 \quad t = 7$

$P = 3200 \quad n = 4$

$$FV = 3200\left(1+\frac{0.09}{4}\right)^{4(7)}$$

$$= 5966.54$$

The future value is \$5966.54.

ex: How much must be invested @ 8% compounded monthly to have \$17,500 in 4 yrs?

$FV = 17500$

$* P =$

$r = 0.08$

$n = 12$

$t = 4$

$$17500 = P\left(1 + \frac{0.08}{12}\right)^{12(4)}$$

$$\Rightarrow P = \frac{17500}{\left(1 + \frac{0.08}{12}\right)^{12(4)}}$$

$$= 12721.11$$

we need to save \$12,721.11 to have \$17,500 in 4 yrs.

ex: How much will Euler have in 1 yr if he invests \$1 @ 100%?

$FV =$

$P = 1$

$r = 1 \leftarrow 100\%$

$n =$

$t = 1$

How is FV impacted by n.

$\underline{n=1:}$

$$FV = 1\left(1 + \frac{1}{1}\right)^{1(1)} = 2$$

$\underline{n=2:}$

$$FV = 1\left(1 + \frac{1}{2}\right)^{2(1)} = 2.25$$

$N=4$:

$FV = (1+\frac{1}{4})^4 = 2.44$

N:

$FV = (1+\frac{1}{N})^N$

As $N \to \infty$ we see $FV \to e$

$\uparrow$

Euler's number.

w/ continuous compounding, the best Euler gets is $FV \approx e$

Formula for continuous compounding.

(3) $FV = Pe^{rt}$.

ex: Find the future value if \$3200 is invested for 7 yrs @ 9% compounded continuously.

*FV

$P = 3200$

$r = 0.09$

$N = N/A$

$t = 7$

$FV = 3200e^{0.09(7)}$

$= 6008.35$

The future value is \$6,008.35.

ex: How much must be invested @ 8% compounded continuously to have \$17500 in 4 yrs?

$FV = 17500$

$*P =$

$r = 0.08$

$n = N/A$

$t = 4$

solve $17500 = Pe^{0.08(4)}$

$\Rightarrow P = \dfrac{17500}{e^{0.08(4)}}$

$= 12707.61$

we need to invest \$12707.61 today.