6.1: Sequences.

Sequence: List of numbers
A fcn whose domain
is 1, 2, 3, 4, ...

Notation.

$$f(x) = 2x - 5$$

$$f(1) = -3;\ f(2) = -1;\ f(3) = 1$$
$$f(\tfrac{1}{2}) = -4$$

$$a_n = 2n - 5$$

$$a_1 = -3;\ a_2 = -1;\ a_3 = 1;\ a_{1/2} \text{ undefined.}$$

ex: write a few terms of

(a) $a_n = 3n + 1$

$$a_1 = 4;\ a_2 = 7;\ a_3 = 10;\ a_4 = 13, \ldots$$

(b) $b_n = n^2 + n$

$$b_1 = 2;\ b_2 = 6;\ b_3 = 12; \ldots$$
$$\{b_n\} = \{2, 6, 12, \ldots\}$$

ex: Cool famous example Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

ratio of consecutive Fibonacci #'s.

$\frac{1}{1} = 1$ $\frac{2}{1} = 2$ $\frac{3}{2} = 1.5$ $\frac{5}{3} = 1.67$

$\frac{8}{5} = 1.6$ $\frac{13}{8} = 1.625$ $\frac{21}{13} \approx 1.615$ $\frac{34}{21} \approx 1.619$

Approach the Golden Ratio $\frac{1+\sqrt{5}}{2}$

2, 7, 9, 16, 25, 41, 66, 107, ...

$\frac{7}{2} = 3.5$ $\frac{9}{7} \approx 1.28$ $\frac{16}{9} = 1.7$ $\frac{25}{16} = 1.56$

$\frac{41}{25} = 1.64$ $\frac{66}{41} = 1.609$ $\frac{107}{66} = 1.621$ ...

also approaches $\frac{1+\sqrt{5}}{2}$.

Arithmetic sequences. (linear)

ex: $a_n = -3n + 4$

$\{a_n\} = \{1, -2, -5, -8, \ldots\}$

common difference $= -3$. (slope)
1st term $a_1 = 1$

Formula: $a_n = a_1 + d(n-1)$

nth term of an arith. seq.

ex: (a) $1, 4, 7, 10, 13, \ldots$

common difference $= 3$
1st term: 1

$\Rightarrow a_n = 1 + 3(n-1)$

check: $a_n = 1 + 3(n-1)$
$= 1 + 3n - 3$
$= 3n - 2$

(b) $15, 8, 1, -6, \ldots$

common difference: $d = -7$

$b_1 = 15$

$b_n = 15 - 7(n-1)$

recall: $a_n = a_1 + d(n-1)$ (1)

n^{th} term of an arithmetic sequence.

ex: Find the 10th term of the arithmetic sequence whose 1st term is -3 & common difference 2.

$a_1 = -3$; $d = 2$

$a_{10} = -3 + (2)(10-1) = 15.$

ex: Find the 40th of the arithmetic seq. whose 4th term is -13 and 62nd term is -361.

process

(i) set up 2 eqns

(ii) solve for a_1 & d.

(iii) find a_{40}.

(i) set up eqns.

$a_4 : -13 = a_1 + d(4-1) = a_1 + 3d.$

$a_{62} = -361 = a_1 + 61d$

$\Rightarrow \begin{cases} a_1 + 3d = -13 \\ a_1 + 61d = -361 \end{cases}$ two eqns and two unknowns

(ii) solve for a_1 & d.

$\Rightarrow -58d = 348$ ← using elimination method.

$\Rightarrow d = -6$

$a_1 + 3d = -13$ ← sub to find a_1

$\Rightarrow a_1 + 3(-6) = -13$

$\Rightarrow a_1 = 5$

(iii) find a_{40}

$$a_{40} = 5 + (-6)(40-1)$$

$$= 5 - 6(39)$$

$$= -229$$

Simple Interest.

ex: How much will we have if we invest $1000 @ 12% simple interest for 5 yrs?

yr	$	
0	1000	
1	1120	= 1000 + 120
2	1240	= 1000 + 120 + 120
		= 1000 + (120)(2)
3	1360	= 1000 + 3(120)
5	1600	= 1000 + 5(120)

We will have $1600 after 5 yrs.

Q: what's the formula.

$$1600 = 1000 + 5(120)$$

$$FV = P + t(1000 \cdot 0.12)$$

FV: future value

P: present value

t: yrs

1000: P

0.12: r rate

$\Rightarrow$ $FV = P + t \cdot P \cdot r$

$FV = P(1 + rt)$ (3)

simple interest formula

ex: How much do we have if \$1500 @ 6% simple interest is invested for 7 yrs?

* FV

$P = 1500$

$r = 0.06$

$t = 7$

$FV = 1500(1 + .06(7))$

$= 1500(1.42)$

$= 2130$

The future value is \$2130.

ex: What simple interest rate must we earn to double \$2000 in 5 yrs?

$FV = 4000$

$P = 2000$

* r

$t = 5$

$4000 = 2000(1 + 5r)$

$\Rightarrow 2 = 1 + 5r$

$\Rightarrow 1 = 5r$

$\Rightarrow r = 1/5 = .2$

We must earn 20% each year

Sum of an arithmetic sequence

ex: $1 + 2 + 3 + 4 + \dots + 99 + 100 = \frac{100(101)}{2}$

$= 5050$

$$\begin{array}{l} 1 + 2 + 3 + \dots + 98 + 99 + 100 \\ + \underline{100 + 99 + 98 + \dots + 3 + 2 + 1} \\ 101 + 101 + 101 + \dots + 101 + 101 + 101 \end{array}$$

$= 100(101)$

ex: $17 + 28 + 39 + \dots + 1502 + 1513$ ← 137th term.

$$+ \underline{1513 + 1502 + 1491 + \dots + 28 + 17}$$

$= 1530 + 1530 + 1530 + \dots + 1530 + 1530$

$= 137(1530)$ (# of terms, 1st + last terms)

and our sum is $\frac{137(1530)}{2} = 104805$

Formula: $S_N = \frac{N(a_1 + a_N)}{2}$ (2)

Sum of the 1st N terms of an arithmetic sequence.