5.2 : Logs.

in math, log means exponent.

ex: solve

a) $2^x = 32 \Rightarrow x = 5$

b) $3^x = 81 \Rightarrow x = 4$

c) $2^x = 7 \Rightarrow x \approx 2.8073549$

we have an unknown exponent ... we need the logarithm (log).

Defn: If $b > 0$ and $b \neq 1$ then $x = b^y \Leftrightarrow y = \log_b(x)$

5.2 cont

Dfn: If $b > 0$ and $b \neq 1$ then

$$\underbrace{x = b^y}_{\text{exponential form}} \iff \underbrace{y = \log_b(x)}_{\text{logarithmic form.}}$$

ex: write in log form

(a) $9 = 3^2 \iff 2 = \log_3(9)$

exponent means log.

(b) $16 = 2^4 \iff 4 = \log_2(16)$

(c) $7^{-2} = \frac{1}{49} \iff -2 = \log_7\left(\frac{1}{49}\right)$

Notation

Dusty

$\log_b(x)$

ex: write in exponential form

(a) $\log_3(81) = 4 \iff 3^4 = 81$

(b) $-1 = \log_e\left(\frac{1}{e}\right) \iff e^{-1} = \frac{1}{e}$

(c) $\log_9(x) = y \iff 9^y = x$

Notation: many logs.

$\log_2(x)$, $\log_5(x)$, $\log_\pi(x)$ ...

→ $\log(x) = \log_{10}(x)$ common log

→ $\ln(x) = \log_e(x)$ natural log.

ex: use calculator

$\log(17) \approx 1.23 \iff 10^{1.23} \approx 17$

$\ln(17) \approx 2.833 \iff e^{2.833} \approx 17$

$\log_5(17) = ?$

ex: solve $m = 20 \ln\left(\frac{40}{40-x}\right)$ for x.

isolate the log.

$$\Rightarrow \frac{m}{20} = \log_e\left(\frac{40}{40-x}\right)$$

log form $\Updownarrow$ exp form

$$\Rightarrow e^{m/20} = \frac{40}{40-x}$$

$$\Rightarrow (40-x)\, e^{m/20} = 40$$

$$\Rightarrow 40 - x = \frac{40}{e^{m/20}}$$

$$\Rightarrow -x = -40 + \frac{40}{e^{m/20}}$$

$$\Rightarrow x = 40 - \frac{40}{e^{m/20}}.$$

Log Properties.

Assume $b > 0$, $b \neq 1$, $m, n > 0$, and $r \in \mathbb{R}$

(I) $\log_b(b^x) = x$

(II) $b^{\log_b(x)} = x$, $x > 0$

$\Big\}$ logs and exp undo eachother.

(III) $\log_b(m \cdot n) = \log_b(m) + \log_b(n)$

The log of a product is the sum of logs.

(IV) $\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$

The log of a quotient is the difference of logs.

(V) $\log_b(m^r) = r \log_b(m)$

The log of a power is the log times the power ex: write as a single log.

(a) $\ln(x) - \ln(y) = \ln\left(\frac{x}{y}\right)$

(b) $\log_3(x+1) + \log_3(x-1) = \log_3\left[(x+1)(x-1)\right]$

(c) $\log(2x+1) - \frac{1}{3}\log(x+1) = \log(2x+1) - \log(x+1)^{1/3}$

$$= \log\left(\frac{2x+1}{(x+1)^{1/3}}\right)$$

ex: write as a sum/difference of logs w/o exponents.

(a) $\log\left(\frac{x-7}{x+6}\right) = \log(x-7) - \log(x+6)$

(b) $\ln\left[(x+1)(4x+5)\right] = \ln(x+1) + \ln(4x+5)$

(c) $\log_2\left(x(x+4)^3\right) = \log_2(x) + \log_2(x+4)^3$

$$= \log_2(x) + 3\log_2(x+4).$$

ex: $\log_5(17) = x$

$\Leftrightarrow \quad 5^x = 17$

$\Leftrightarrow \quad \log(5^x) = \log(17)$

$\Leftrightarrow \quad x\log(5) = \log(17)$

$\Leftrightarrow \quad x = \dfrac{\log(17)}{\log(5)}$

≈ 1.76

check: $5^{1.76} \approx 17$

change of base

$$\log_b(a) = \frac{\log(a)}{\log(b)} = \frac{\ln(a)}{\ln(b)}$$