5.1: Exponential Functions

compare $y = x^2$ w/ $y = 2^x$.

exponentials grow faster than polynomials

compare $y = 2x$ w/ $y = 2^x$

x	$y = 2x$	
-2	-4	
		$+2$
-1	-2	
		$+2$
0	0	
		$+2$
1	2	
		$+2$
2	4	
		$+2$
3	6	

constant growth rate.

x	$y = 2^x$	
-2	$1/4$	
		$*2$
-1	$1/2$	
		$*2$
0	1	
		$*2$
1	2	
		$*2$
2	4	
		$*2$
3	8	

constant growth factor.

compare $y = 2x + 3$ w/ $y = 3 \cdot 2^x$

x	$y = 2x + 3$	
-2	-1	$> +2$
-1	1	$> +2$
0	3	$> +2$
1	5	
2	7	
3	9	

" $+2$ "

y-int = 3

x	$y = 3 \cdot 2^x$	
-2	$3/4$	
-1	$3/2$	$> * 2$
0	3	$> * 2$
1	6	
2	12	$> * 2$
3	24	

" $*2$ "

y-int = 3

$y = mx + c$

m = slope (growth rate)

c = y-int

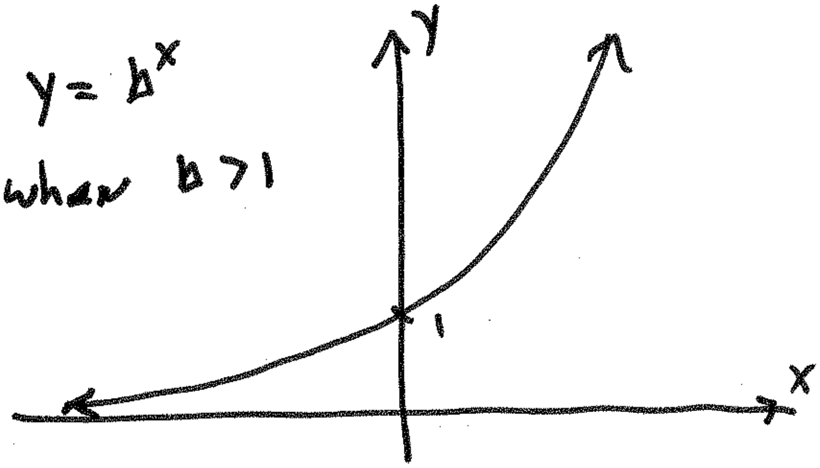
$y = a \cdot b^x$

b = growth factor

a = y-int.

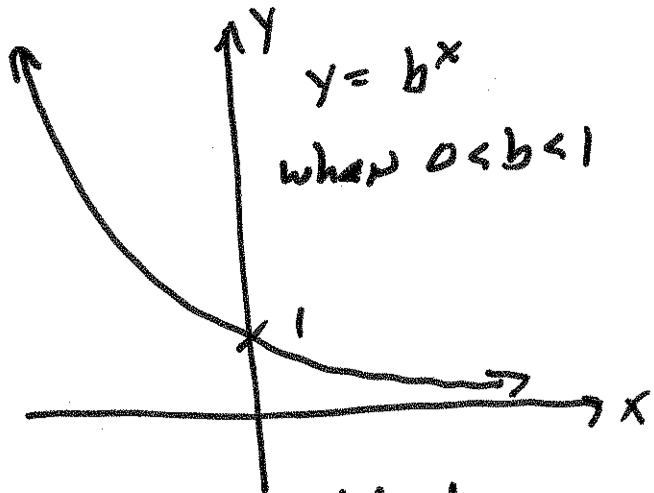
Basic shape.

$y = b^x$
when $b > 1$



exponential growth

$y = b^x$
when $0 < b < 1$



exponential decay.

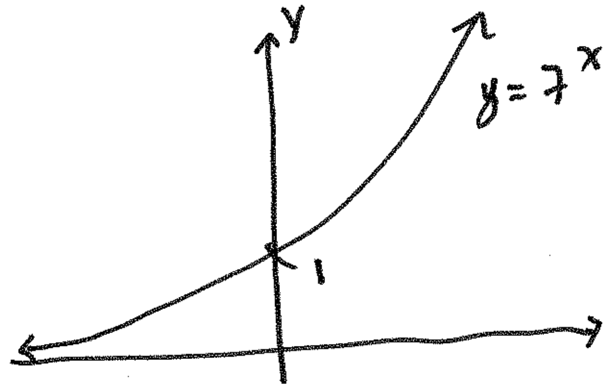
y-int = 1 (or if $y = a \cdot b^x$, the y-int @ a).

b = growth factor

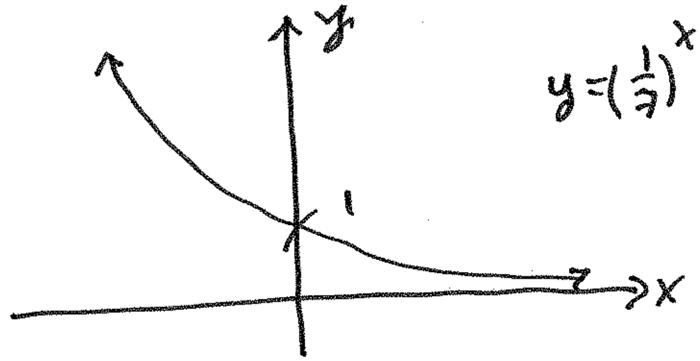
Domain: $\mathbb{R}$
Range: $(0, \infty)$
horizontal asymptote @ $y=0$.

ex: sketch

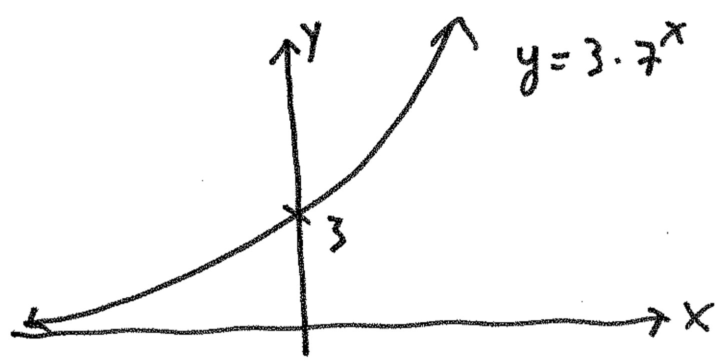
(a) $y = 7^x$



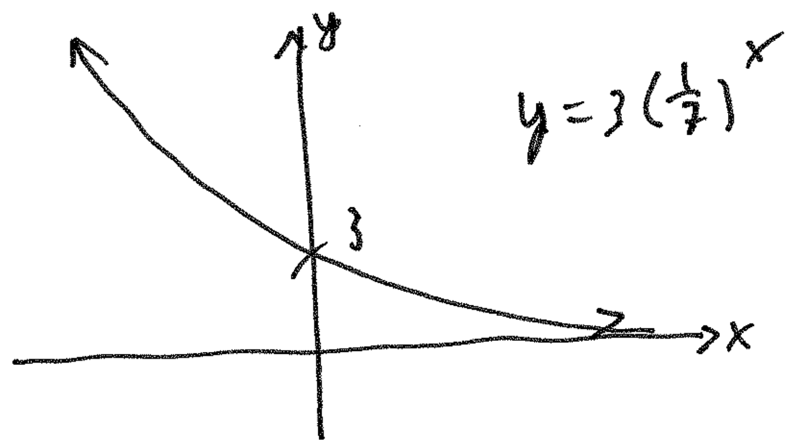
(b) $y = \left(\frac{1}{7}\right)^x$



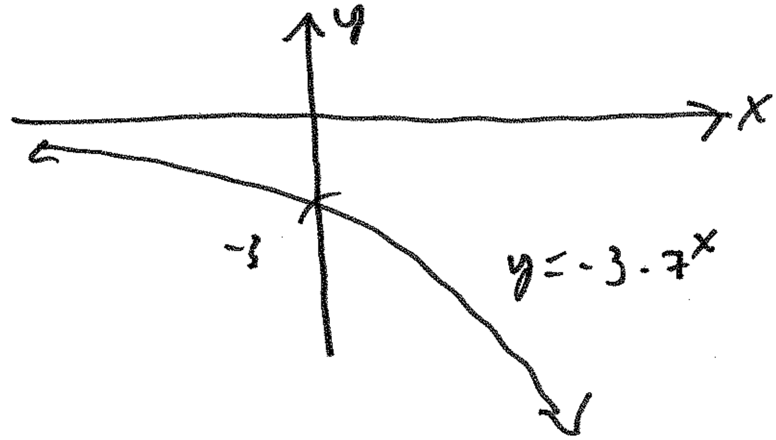
(c) $y = 3 \cdot 7^x$



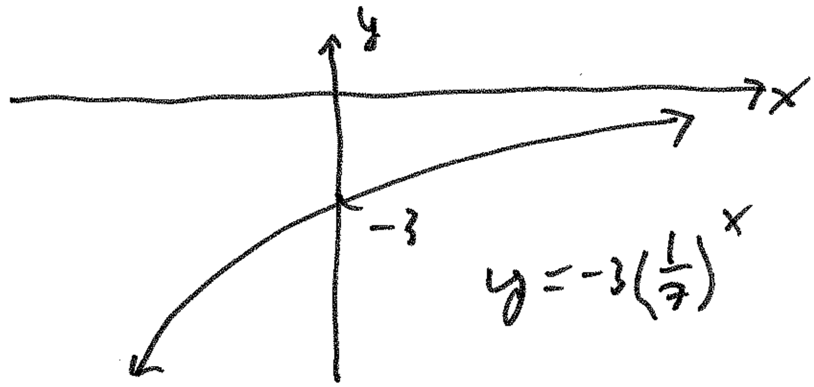
(d) $y = 3 \cdot \left(\frac{1}{7}\right)^x$



(e) $y = -3 \cdot 7^x$



(f) $y = -3 \cdot \left(\frac{1}{7}\right)^x$



Euler's Number: e

$$e = \left(1 + \frac{1}{x}\right)^x \text{ as } x \to \infty.$$

Huh?

Tug of war.

$1^5 = 1$

$1^4 = 1$

$1^3 = 1$

$1^2 = 1$

$1^1 = 1$

x	$1 + \frac{1}{x}$
1	$1 + \frac{1}{1} = 2$
10	$1 + \frac{1}{10} = 1.1$
100	$1 + \frac{1}{100} = 1.01$
$\downarrow$	$\downarrow$
∞	1

$5^\infty = \infty$

$4^\infty = \infty$

$3^\infty = \infty$

$2^\infty = \infty$

$1^\infty = ?$

As $x \to \infty$ $\left(1+\frac{1}{x}\right)^x \not\to 1^\infty$

$\left(1+\frac{1}{x}\right)^x \to e$

$e \approx 2.71828\ldots$

Euler's number : e

$e \approx 2.718281828$

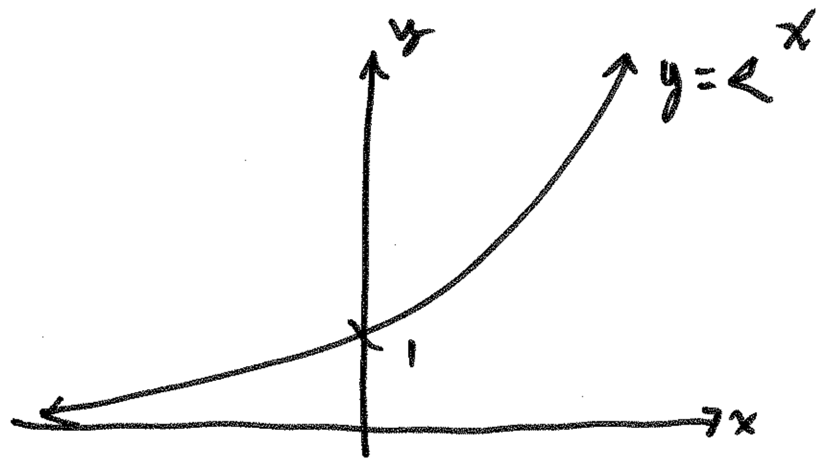
e is irrational. It can't be written as the fraction/ratio of integers.

Q: Is $e = \frac{2,718,281,828}{1,000,000,000}$?

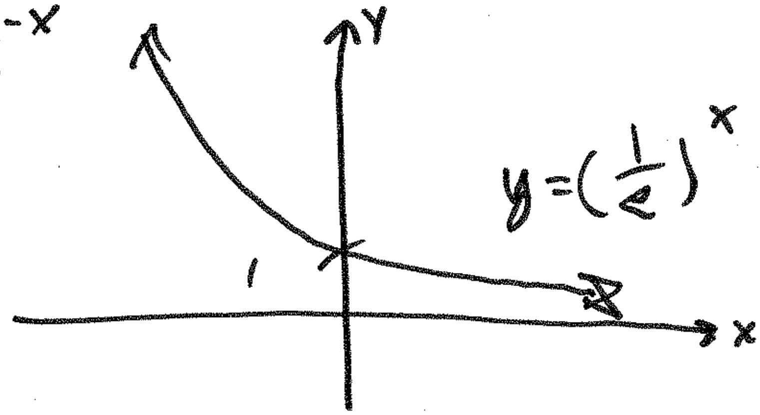
No, e is irrational.

ex: sketch

(a) $y = e^x$



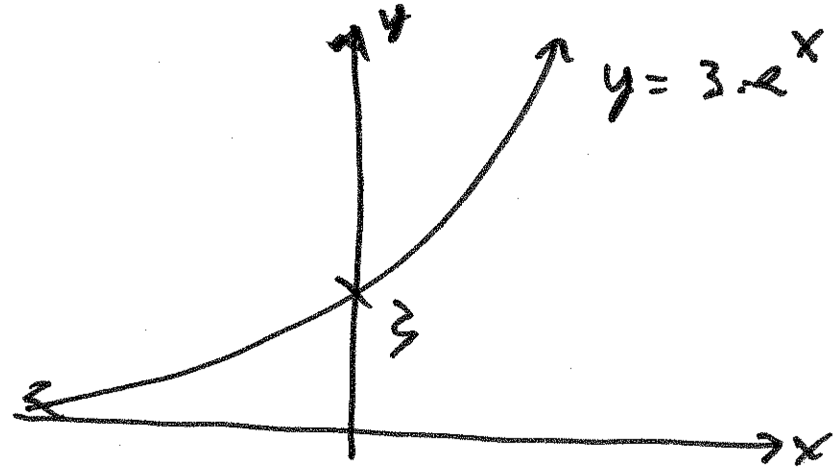
(b) $y = \left(\frac{1}{e}\right)^x = e^{-x}$



c) $y = 3 \cdot e^x$

Domain: $\mathbb{R}$

Range: $(0, \infty)$



Pattern in growth rates.

(1) we have 9.5% sales tax

I buy a \$10 item.

tax: $10 \cdot 0.095 = 0.95$

price 10.00

total 10.95

calculate $10(1.095) = 10.95$

$1 + 0.095$

100% tax

(2) Tax rate = 12%

item cost $\$50$

Total cost $= 50(\underset{\substack{\uparrow \\ 100\%}}{1} + \underset{\substack{\uparrow \\ \text{tax.}}}{0.12})$

$= 56$

(3) 20% off sale on a $\$40$ item.

price = 40

savings = $40(.2) = 8$

total = 32

Calculate $40(\underset{\substack{\uparrow \\ 1 - 0.2}}{0.8}) = 32$

$$\underset{\substack{\uparrow \\ 100\%}}{1} - \underset{\substack{\uparrow \\ 20\% \\ \text{savings.}}}{0.2}$$

(4) Interpret.

(a) $30(1.07) \leftarrow 1.07 = 1 + 0.07$

starting amt: 30

grows by 7%

(b) $50(0.60) \longleftarrow 0.6 = 1 - .40$

starting amt : 50

decrease by 40%

modeling populations.

(a) Ethiopia – 90.2 mil

2.1% growth.

$E(t) = 90.2(1 + 0.021)^t$

$= 90.2(1.021)^t$

(b) Iraq – 35.9 mil

3.18% growth

$I(t) = 35.9(1.0318)^t$

$\uparrow$

$1 + 0.0318$

(c) Germany – 81.6 mil

0.228% decline

$G(t) = 81.6(1 - 0.00228)^t$

$= 81.6(0.99772)^t$