3.3: Gauss-Jordan Elimination

Three ops.

stand up / sit down exercise
to teach order

Augmented matrix

$$\left[\begin{array}{cc|c} 2 & -1 & 5 \\ 1 & 3 & 6 \end{array}\right] \quad R_1 \leftrightarrow R_2 \quad \text{Row swap}$$

$$\left[\begin{array}{cc|c} 1 & 3 & 6 \\ 2 & -1 & 5 \end{array}\right] \quad R_2 - 2R_1 \to R_2 \quad \text{Add Rows}$$

$$\left[\begin{array}{cc|c} 1 & 3 & 6 \\ 0 & -7 & -7 \end{array}\right] \quad -\tfrac{1}{7}R_2 \to R_2$$

multiplied a row by a scalar (numb

$$\left[\begin{array}{cc|c} 1 & 3 & 6 \\ 0 & 1 & 1 \end{array}\right] \quad R_1 - 3R_2 \to R_1 \quad \text{Add rows}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \end{array}\right]$$

$\Rightarrow 1x + 0y = 3$ and $0x + 1y = 1$

$\Rightarrow x = 3$ and $y = 1$.

3 Row operations.

$$\left.\begin{array}{l}\text{(1) row swap}\\ \text{(2) mult. row by a scalar}\end{array}\right\}\ \text{To get a 1}$$

$$\left.\text{(3) add rows.}\right\}\ \text{To get a 0.}$$

In 2eqt case

$$\left[\begin{array}{cc|c} \sim & \sim & \sim \\ \sim & \sim & \sim \end{array}\right] \xrightarrow{\text{row swap or mult. by const.}} \left[\begin{array}{cc|c} 1 & \sim & \sim \\ \sim & \sim & \sim \end{array}\right]$$

$$\xrightarrow{\text{Add rows.}} \left[\begin{array}{cc|c} 1 & \sim & \sim \\ 0 & \sim & \sim \end{array}\right] \xrightarrow{\text{mult. by constant}} \left[\begin{array}{cc|c} 1 & \sim & \sim \\ 0 & 1 & \sim \end{array}\right] \xrightarrow{\text{Add rows.}} \left[\begin{array}{cc|c} 1 & 0 & \sim \\ 0 & 1 & \sim \end{array}\right]$$

Basic Row Operations.

(1) swap rows

(2) multiply row by a scalar.

$\Big\}$ To get a leading 1

(3) add rows. } To get 0.

ex:

$$\begin{cases} 2x + y = 5 \\ 3x + 2y = 8 \end{cases}$$

$$\Rightarrow \left[\begin{array}{cc|c} 2 & 1 & 5 \\ 3 & 2 & 8 \end{array}\right] \quad \tfrac{1}{2}R_1 \to R_1$$

$$\Rightarrow \left[\begin{array}{ccc} 1 & 1/2 & 5/2 \\ 3 & 2 & 8 \end{array}\right] \quad R_2 - 3R_1 \to R_2$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 1/2 & 5/2 \\ 0 & 1/2 & 1/2 \end{array}\right] \quad 2R_2 \to R_2$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 1/2 & 5/2 \\ 0 & 1 & 1 \end{array}\right] \quad R_1 - \tfrac{1}{2}R_2 \to R_1$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

$x = 2$ & $y = 1$

$\boxed{\begin{matrix} 3.3 \\ 5/8 \end{matrix}}$

ex: solve $\begin{cases} x + 2y - z = 3 \\ 2x + 5y - 2z = 7 \\ -x + y + 5z = -12 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 5 & -2 & 7 \\ -1 & 1 & 5 & -12 \end{array}\right] \quad R_2 - 2R_1 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ -1 & 1 & 5 & -12 \end{array}\right] \quad R_3 + 1R_1 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 3 & 4 & -9 \end{array}\right] \quad R_3 - 3R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 4 & -12 \end{array}\right] \quad \tfrac{1}{4} R_3 \longmapsto R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right] \quad R_1 + 1R_3 \to R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right] \quad R_1 - 2R_2 \to R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right] \qquad \begin{aligned} x &= -2 \\ y &= 1 \\ z &= -3 \end{aligned}$$

ex:

$$\begin{cases} 2x + 3y + 4z = 2 \\ x + 2y + 2z = 1 \\ x + y + 2z = 2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 2 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 2 & 2 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 2 & 3 & 4 & 2 \\ 1 & 1 & 2 & 2 \end{array}\right] \quad R_2 - 2R_1 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 1 & 2 & 2 \end{array}\right] \quad R_3 - R_1 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] \quad -R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] \quad R_3 + R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$0x + 0y + 0z = 1$ False

"no solution"

3.3

7/8

ex:

$$\begin{cases} 2x + 3y + 4z = 9 \\ x + y + z = 3 \\ 3x + 4y + 5z = 12 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 9 \\ 1 & 1 & 1 & 3 \\ 3 & 4 & 5 & 12 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 3 & 4 & 9 \\ 3 & 4 & 5 & 12 \end{array}\right] \quad R_2 - 2R_1 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 5 & 12 \end{array}\right] \quad R_3 - 3R_1 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{array}\right] \quad R_3 - R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_1 - R_2 \to R_1$$

$0x + 0y + 0z = 0$ True.

"infinite # of solutions"

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \to x - z = 0 \\ \to y + 2z = 3 \\ \end{array}$$

$x = z$

$y = 3 - 2z$

$z =$ anything

ex: $\left\{ \text{Junk} \right.$

$$\left[\begin{array}{c|} \text{Junk} \\ \text{coefficients} \end{array} \right]$$

⋮ row reduction

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \longrightarrow x + 4z = 2 \\ \longrightarrow y + z = 3 \\ \end{array}$$

$x = 2 - 4z$

$y = 3 - z$

$z =$ anything.

There are infinitely many sol'ns...

three are:

$x = 2$	$x = -2$	$x = -18$
$y = 3$	$y = 2$	$y = -2$
$z = 0$	$z = 1$	$z = 5$