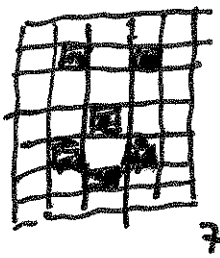
3.1: Intro to matrices.



7x5

1 = black pixel
0 = white pixel.

Matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{7\times 5}$$

rows (7)

columns (5)

Dimension.

Vocab: matrix ← singular

matrices ← plural.

rows & cols.

ex: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$

2 rows

3 cols

#'s inside a matrix: element/entry.

$a_{21} = 4$; $a_{13} = 3$; $a_{22} = 5$

ambiguous: b_{1234} ← we won't do this.

3.1
2/4

ex: $C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{3\times 4}$ zero matrix.

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

In order for matrices to be equal

(1) same dimensions

(2) same entries.

ex: solve

$$\begin{bmatrix} x & 3 & (2x-1) \\ y & 4 & 4y \end{bmatrix} = \begin{bmatrix} (2x-4) & z & 7 \\ 1 & (w+1) & (3y+1) \end{bmatrix}$$

$x = 2x-4$

$\Rightarrow \boxed{4 = x}$

$\boxed{3 = z}$

$2x-1 = 7$ ✓

$\boxed{y = 1}$

$4 = w+1$

$\Rightarrow \boxed{3 = w}$

$4y = 3y+1$ ✓

soln: $x = 4$

$y = 1$

$z = 3$

$w = 3$

Transpose: swap rows & cols.

ex: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Identity matrix

$I_1 = [1]$

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Algebra of matrices.

adding matrices

mult. of matrices
- scalar mult. mult. # and a matrix
- matrix mult. mult. 2 matrices

ex: Let $C = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$ & $D = \begin{bmatrix} 11 & -7 \\ -3 & -5 \end{bmatrix}$

(a) $C + D = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 11 & -7 \\ -3 & -5 \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ 0 & -6 \end{bmatrix}$

(b) $\frac{1}{2}C = \frac{1}{2}\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 3/2 & -1/2 \end{bmatrix}$

scalar multiplication

(c) $C - D = C + (-1)D$

$= \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} - \begin{bmatrix} 11 & -7 \\ -3 & -5 \end{bmatrix} = \begin{bmatrix} -13 & 11 \\ 6 & 4 \end{bmatrix}$

(d) $2C + 3D = 2\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} + 3\begin{bmatrix} 11 & -7 \\ -3 & -5 \end{bmatrix} = \begin{bmatrix} 29 & -13 \\ -3 & -17 \end{bmatrix}$