2.3: Applications of Quadratics to Business
Math 111

Objective:

- Profit, Revenue, and Cost with **Quadratics**

TOOLS FOR CONSTRUCTING

Revenue = (selling price) $\cdot\ x$ where x is the number of units sold

Costs = (variable cost) $\cdot\ x$ **+ fixed cost** where x is the number of units produced

PROFIT = (Revenue) – (Cost)

COMPETITIVE MARKET
coffee stand
↳ price determined by market

MONOPOLY MARKET
miracle medicine.
↳ price determined by company.

TOOLS FOR ANALYZING

MAXIMIZE, use VERTEX $x = -\frac{b}{2a}$ is the units, then evaluate the function to find max $$$.

BREAKEVEN: R = C

R – C = 0 then use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. In a monopoly market, the demand for a product is $p = 1600 - x$ where x is the number of units sold.

a. Find the Revenue function

Demand.

$R(x) = p \cdot x = (1600 - x) \cdot x = 1600x - x^2.$

b. Find the maximum Revenue and the number of units sold that will maximize Revenue.

$x = -\frac{b}{2a} = -\frac{1600}{2(-1)} = 800.$

$R(800) = 640{,}000$

The max revenue is $640,000 when 800 units are sold.

c. Find the price that will maximize revenue.

$p = 1600 - x$ @ $x = 800$

$\Rightarrow p = 800.$

The price that maximizes revenue is $800/unit.

2. Consider the revenue and cost functions given below.

$$C(x) = 2000 + 40x + x^2$$
$$R(x) = 130x$$

a. Is this a monopoly or competitive market?

price/unit fixed @ \$130.

b. Find the Profit function.

$P(x) = R(x) - C(x)$

$= 130x - (2000 + 40x + x^2)$

$\Rightarrow P(x) = -x^2 + 90x - 2000.$

c. Find and interpret the maximum profit

$x = \frac{-90}{2(-1)} = 45$

$P(45) = 25$

When 45 units are sold, we get our max profit of \$25.

d. Find and interpret the breakeven points

$P(x) = 0$

$\Rightarrow 0 = -x^2 + 90x - 2000$

$\Rightarrow x = \frac{-90 \pm \sqrt{8100 - 4(-1)(-2000)}}{2(-1)}$

$\Rightarrow x = \frac{-90 \pm \sqrt{100}}{-2}$

$x = 40$ OR $x = 50$

The company breaks even when 40 OR 50 units are sold.

3. In a monopoly market, the demand function is $p = 500 - 2x$. Fixed costs are 3600 and variable costs are given by $100 + 2x$

a. Find and interpret the maximum Profit and the number of units produced and sold that will maximize Profit

$R(x) = p \cdot x = (500 - 2x)x = 500x - 2x^2$

$C(x) = (100 + 2x)x + 3600 = 2x^2 + 100x + 3600$

$P(x) = R(x) - C(x) = 500x - 2x^2 - (2x^2 + 100x + 3600)$

$P(x) = -4x^2 + 400x - 3600.$

vertex

$x = \frac{-400}{2(-4)} = 50$

$y = 6400$

when you sell 50 units you get the maximum profit of \$6400.

b. Find and interpret the breakeven points.

solve $P(x) = 0$

$\Rightarrow -4x^2 + 400x - 3600 = 0$

$\Rightarrow x = \frac{-400 \pm \sqrt{160000 - 4(-4)(-3600)}}{2(-4)}$

$\Rightarrow x = 10$ or $x = 90$

The company breaks even when they sell 10 or 90 units.

c. Find and interpret the selling price that will maximize profits.

price: $p = 500 - 2x$ ← $x = 50$

$= 400$

Sell units @ \$400 each too max profit.

ex4: Find equilibrium given

supply $S: p = \dfrac{q+50}{2}$ and

Demand $D: p = \dfrac{100+20q}{q}$

Solve $\dfrac{q+50}{2} = \dfrac{100+20q}{q}$

$\Rightarrow q(q+50) = 2(100+20q)$

$\Rightarrow q^2 + 50q = 200 + 40q.$

$\Rightarrow q^2 + 10q - 200 = 0$

$\Rightarrow (q-10)(q+20)$

$\Rightarrow q = 10$ OR $\cancel{q = -20}$.

and $p = \dfrac{10+50}{2} = 30.$

Equilibrium is reached when 10 units are sold for \$30 ea.

ex5: Find equilibrium given a tax of $12.50 is placed on the supplier & then passed on to the consumer.

$$\text{before tax.}\ \begin{cases} S: p = \dfrac{q+50}{2} \\[2ex] D: p = \dfrac{100+20q}{q} \end{cases}$$

$$\text{after tax}: S_{\text{after}}: p = \frac{q+50}{2} + \underset{\substack{\uparrow \\ \text{Tax}}}{12.50}$$

Solve $\dfrac{q+50}{2} + 12.5 = \dfrac{100+20q}{q}$

$$\Rightarrow 2q\left(\frac{q+50}{2} + 12.5\right) = 2q\left(\frac{100+20q}{q}\right)$$

$$\Rightarrow q^2 + 50q + \underline{25q} = 200 + 40q$$

$$\Rightarrow q^2 + 35q - 200 = 0$$

$$\Rightarrow (q-5)(q+40) = 0$$

$$\Rightarrow q = 5 \text{ OR } \cancel{q = -40}$$

$$\Rightarrow p = \frac{100+20(5)}{5} = 40$$

After tax the price is $40 and 5 are sold at equilibrium.