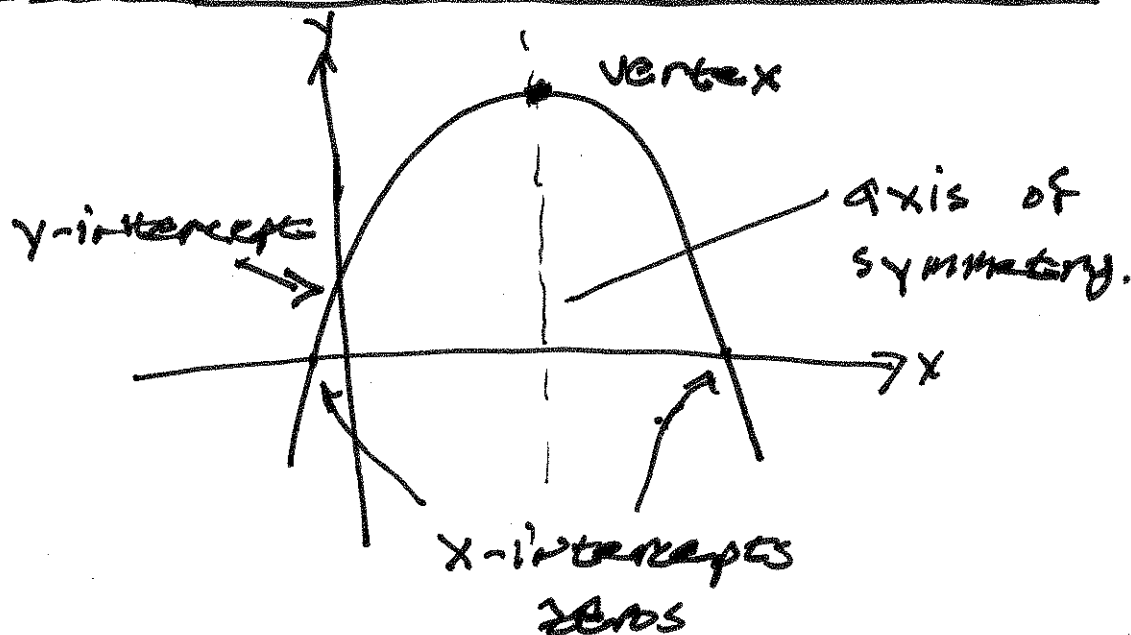


2.2: Graphs of Quadratic Equations.



$$x\text{-intercept(s)}: x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{axis of symmetry: } x = \frac{-b}{2a}$$

$$\text{vertex: } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right)$$

$$\text{where } f(x) = ax^2 + bx + c.$$

ex: Find the vertex of $f(x) = 6 - 4x - 2x^2$

$$a = -2; b = -4; c = 6$$

$$x = \frac{-b}{2a} = -\frac{(-4)}{2(-2)} = -1$$

$$y = f(-1) = 6 - 4(-1) - 2(-1)^2 = 8$$

$$\text{vertex: } (-1, 8)$$

Summary of formulas

If $f(x) = ax^2 + bx + c$

zeros (x-ints): $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

axis of symmetry: $x = -\frac{b}{2a}$

vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

y-int: $y = c$

Shape:



$a > 0$
min



$a < 0$
max

Domain: \mathbb{R}

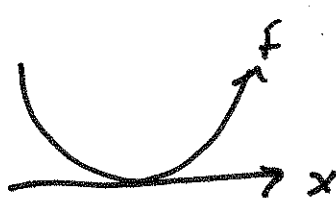
\mathbb{R}

Range: $[\text{min}, \infty)$ $(-\infty, \text{max}]$

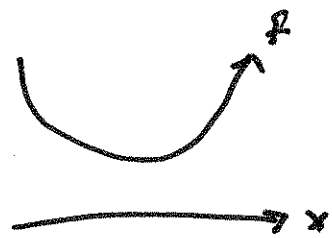
Number of x-intercepts



2 zeros
 $b^2 - 4ac > 0$



1 zero
 $b^2 - 4ac = 0$



No real
zeros.
 $b^2 - 4ac < 0$

ex: Find the vertex of $f(x) = 4 + 3x - x^2$

$$x = -\frac{b}{2a} = -\frac{3}{2(-1)} = \frac{3}{2}$$

$$y = f\left(\frac{3}{2}\right) = 6.75$$

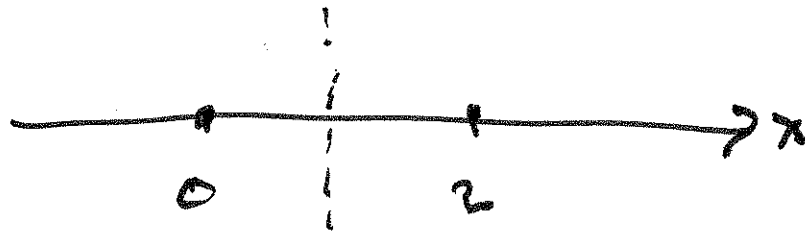
$$\text{vertex: } \left(\frac{3}{2}, \frac{25}{4}\right)$$

ex: Find the zeros & x-int of

$$y = x^2 - 2x$$

$$= x(x - 2)$$

\Rightarrow zeros @ $x = 2$ or $x = 0$



axis of
sym @ $x = 1$

\Rightarrow vertex

$$\left. \begin{array}{l} x = 1 \\ y = -1 \end{array} \right\} (1, -1)$$

ex: Find the zeros & vertex of

$$y = x^2 + x + 2y - 5$$

$$\Rightarrow -y = x^2 + x - 5$$

$$\Rightarrow y = -x^2 - x + 5$$

$$\begin{aligned}\text{zeros: } x &= \frac{1 \pm \sqrt{1 - 4(-1)(5)}}{2(-1)} \\ &= \frac{1 \pm \sqrt{21}}{-2}\end{aligned}$$

vertex:

$$\left. \begin{array}{l} x = \frac{1}{-2} \\ y = +5.25 \end{array} \right\} \left(-\frac{1}{2}, \frac{21}{4} \right)$$

ex: use the calculator to find the zeros & vertex of

$$y = 0.1x^2 + 0.4x - 3.2$$

$$\text{zeros: } x = -8 \text{ \& } x = 4$$

$$\text{vertex: } (-2, -3.6)$$

ex: Suppose the daily profit from the sale of a product is given by $P(x) = 16x - 0.1x^2 - 100$.

(a) find ϵ interp $P(0)$.

$$P(0) = -100.$$

The fixed costs are \$100.

(b) find ϵ interp the vertex

$$x = -\frac{16}{2(-0.1)} = 80$$

$$y = 540$$

The max profit of \$540 is hit w/ the sale of 80 units.