

1.6: Applications of Functions in Business and Economics Math III

Objective:

1. Supply, demand and market equilibrium
2. Monopoly market

1. Supply, Demand and Market Equilibrium

Market equilibrium occurs when the quantity of a commodity demanded is equal to the quantity supplied.

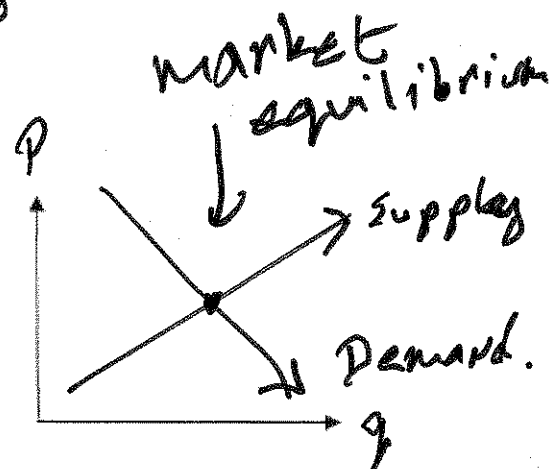
Do you think there is a relationship between demand and price? If so what is it?

price \uparrow quantity demanded \downarrow

Do you think there is a relationship between supply and price? If so what is it?

price \uparrow quantity supplied \uparrow

If the supply and demand curves for a commodity are graphed on the same coordinate system, with the same units, market equilibrium occurs at the point where the curves intersect. The price at that point is the equilibrium price, and the quantity at that point is the equilibrium quantity.



Ex1: Find the market equilibrium for the following supply and demand functions. Explain what it means in everyday language.

Demand: $p = -3q + 36$

Supply: $p = 4q + 1$

set supply equal to demand.

solve $-3q + 36 = 4q + 1$

$\Rightarrow 35 = 7q$

$\Rightarrow q = 5$

and $p = -3(5) + 36$
 $= 21$

Market equilibrium is reached when 5 units are sold for \$21 each.

Ex2: Find the market equilibrium for the following supply and demand functions. Explain what it means in everyday language.

Demand: $(p+2)q = 2100$

Supply: $4p - q = 42 \Rightarrow q = 4p - 42$

and solve $(p+2)(4p-42) = 2100$

$\Rightarrow 4p^2 - 42p + 8p - 84 = 2100$

$\Rightarrow 4p^2 - 34p - 2184 = 0$

find x-intercepts of

$y = 4x^2 - 34x - 2184$

~~$x = -19.5$~~ or $x = 28$
 \uparrow price \uparrow price

$\Rightarrow q = 4(28) - 42$
 $= 70$

Market equilibrium is reached when 70 units are sold for \$28 each.

II. Revenue, Cost, Profit, and Breakeven Analysis

What is Revenue (in a business context)?

money that comes in.

What is Cost (in a business context) and what are its two components?

money spent (that goes out).

What is Profit?

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

Ex3: A manufacturer has fixed costs of \$3300 and a variable cost of \$5 per item produced.

a.) Define your variables

$N = \#$ of units produced. ~~Fixed costs.~~ costs to keep the lights on.

$C =$ cost to produce.

b.) What is the cost function? N items.

N	C
0	3300
1	3305
2	3310

$$C(N) = 3300 + 5N$$

c.) What is the cost if no items are produced?

$$C(0) = 3300 \leftarrow \text{Fixed costs.}$$

Ex4: Heavenly Cappuccino has costs of \$3000/mo if they make and sell 1000 drinks and \$4200/mo if they produce 2000 drinks. Assuming the costs vary in a linear manner:

a.) Define your variables

$n = \# \text{ of drinks}$

$C = \text{cost to produce } n \text{ drinks}$

b.) Find and interpret the slope.

(n, C)

$(1000, 3000)$

$(2000, 4200)$

slope: $m = \frac{4200 - 3000}{2000 - 1000}$
 $= 1.2$

c.) Find and interpret the C-intercept $\leftarrow n=0$

$C\text{-int} = 1800$

The fixed costs are \$1800

d.) Find an equation to model the costs.

$$C - 3000 = 1.2(n - 1000)$$

$$= 1.2n - 1200$$

$\Rightarrow C = 1.2n + 1800$

e.) Find and interpret the n-intercept $\leftarrow C=0$

Solve $0 = 1.2n + 1800$

$\Rightarrow -1800 = 1.2n$

$\Rightarrow n = \frac{-1800}{1.2} = -1500$

There were \$0 costs when -1500 drinks were produced.

$$C = 1.2n + 1800$$

Ex5: The cost function for Heavenly Cappuccino was found in the previous example. If the average drink sells for \$3.50, how many drinks must they sell to breakeven? ← REVENUE = COST.

n	R
0	0
1	3.5
2	7
3	10.5

$$R(n) = 3.5n$$

$$\text{solve: } 3.5n = 1.2n + 1800$$

$$\Rightarrow 2.3n = 1800$$

$$\Rightarrow n = \frac{1800}{2.3} \approx 782.6$$

They must sell ~~783~~ drinks to break

Ex6: Suppose the manufacturer from (ex 3) has revenue $R(n) = 385n$. even

a.) Find the profit function.

$$\text{profit} = \text{REVENUE} - \text{COST}$$

$$P = R - C = 385n - (3300 + 5n)$$

$$= 380n - 3300$$

b.) Find and interpret \overline{MP}

\overline{MP} marginal profit. (slope)

$\overline{MP} = 380$ for each item sold profit increases \$380.

c.) What is the profit on 351 items?

$$P(351) = 380(351) - 3300$$

$$= 130,080$$

IF they sell 351 items the profit

d.) How many items must be sold to breakeven?

$$\text{solve } 0 = 380n - 3300$$

$$\Rightarrow 3300 = 380n$$

$$\Rightarrow n = \frac{3300}{380} \approx 8.7$$

They must sell 9 units to break even

is \$130,080.

break even

$R = C$

$P = 0$

Ex7: The bookstore's quarterly costs are given by $C(n) = 40n + 12000$.

If they wish to breakeven after selling only 200 texts, what must be the price of the average textbook?

$$C(n) = \underbrace{40n}_{\substack{\text{Marginal} \\ \text{cost } MC}} + \underbrace{12000}_{\substack{\text{Fixed} \\ \text{costs}}}$$

breakeven: $\left(\text{Cost of } \underbrace{200 \text{ books}} \right) = \left(\text{Revenue from } \underbrace{200 \text{ books}} \right)$

$$C(200) = 40(200) + 12000$$

\Rightarrow solve: $20000 = \text{revenue}$.

$$= 200 \cdot \left(\begin{array}{l} \text{average} \\ \text{selling} \\ \text{price} \end{array} \right)$$

\uparrow
of books

$$\Rightarrow \text{average selling price} = \frac{20000}{200} = 100$$

The ave. book must sell for \$100 to break even after selling 200 books.