Test 1
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Math 153

No work = no credit

No Symbolic Calculators

Name: KEY

I myself, a professional mathematician, on re-reading my own work find it strains my mental powers to recall to mind from the figures the meanings of the demonstrations, meanings which I myself originally put into the figures and the text from my mind. But when I attempt to remedy the obscurity of the material by putting in extra words, I see myself falling into the opposite fault of becoming chatty in something mathematical.

Johannes Kepler (1597 - 1630)
German astronomer

Warm-ups (1 pt each): $\vec{i}\cdot\vec{j} = 0$ $\quad \vec{j}\times\vec{i} = -\vec{k}$ $\quad \vec{i}-\vec{j} = \langle 1,-1,0\rangle$

1.) (1 pt) Based upon the quote above, how did easily did Kepler understand his earlier work? Answer using complete English sentences.

It was difficult.

2.) (12 pts) Consider the plane $x+2y+3z=4$

a.) Find three points on the plane (not co-linear)

$(4, 0, 0)$
$(2, 1, 0)$
$(1, 0, 1)$

b.) Find the distance from the plane to the point $A(4,5,6)$.

$$D = \frac{|1\cdot 4 + 2\cdot 5 + 3\cdot 6 - 4|}{\sqrt{1^2+2^2+3^2}} = \frac{28}{\sqrt{14}}$$

c.) Find the equation of the line that is normal to the plane through point A. Give your answer parametrically.

$$\vec{r}(t) = \langle 4, 5, 6\rangle + \langle 1, 2, 3\rangle t$$

3.) (12 pts) Consider the two planes $x+y+z=1$ and $x+y=2$.

a.) Find the angle between the two planes.

$$\langle 1,1,1\rangle \cdot \langle 1,1,0\rangle = \sqrt{3}\cdot\sqrt{2}\cos\theta$$

$$\Rightarrow \quad \frac{2}{\sqrt{6}} = \cos\theta$$

0.615 rad

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$$

35.264°

b.) Find the equation of the line where the two planes intersect. Give your answer parametrically.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \langle -1, 1, 0\rangle$$

pt in common $(1,1,-1)$

$$\vec{r}(t) = \langle 1,1,-1\rangle + \langle -1,1,0\rangle t$$

4.) (12 pts) Use the arclength formula to verify that the circumference of a circle with radius R is $2\pi R$. Begin by writing a parametric equation for a circle of radius R centered at the origin.

$$\vec{r}(\theta) = \langle R\cos\theta, R\sin\theta\rangle$$

$$\Rightarrow \vec{r}\,'(\theta) = \langle -R\sin\theta, R\cos\theta\rangle$$

$$\Rightarrow |\vec{r}\,'(\theta)| = R$$

5 pts if a parametric eqn. but not a circle.

$$s = \int_0^{2\pi} R\, d\theta$$

$$= 2\pi R.$$

6.) Write $\vec{a}$ in the form $\vec{a} = a_T\vec{T} + a_N\vec{N}$ without finding $\vec{T}$ and $\vec{N}$ for the position vector-valued function $\vec{r}(t) = t^2\vec{i} + \left(t + \frac{t^3}{3}\right)\vec{j} + \left(t - \frac{t^3}{3}\right)\vec{k}$ at $t = 2$. That is, find $a_T(2)$ & $a_N(2)$.

$$\vec{r}'(t) = \langle 2t, 1+t^2, 1-t^2 \rangle \Big|_{t=2} \quad \langle 4, 5, -3 \rangle$$

$$\vec{r}''(t) = \langle 2, 0 + 2t, -2t \rangle \Big|_{t=2} \quad \langle 2, 4, -4 \rangle$$

$$|\vec{r}'(2)| = \sqrt{16 + 25 + 9} = \sqrt{50}$$

$$a_T = \frac{8 + 20 + 12}{5\sqrt{2}} = \frac{40}{5\sqrt{2}} = \frac{8}{\sqrt{2}}$$

$$(\vec{r}' \times \vec{r}'')(2) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 5 & -3 \\ 2 & 4 & -4 \end{vmatrix} = \langle -8, 10, 6 \rangle$$

w/ magnitude $\sqrt{64 + 100 + 36}$

$= 10\sqrt{2}$

$$a_N = \frac{10\sqrt{2}}{5\sqrt{2}} = 2$$

So $\vec{a} = \frac{8}{\sqrt{2}}\vec{T} + 2\vec{N}$ when $t = 2$.

6.) (15 pts) Find $\vec{T}, \vec{N}, \vec{B}$, and κ for $\vec{r}(t) = \langle 3\cos t, 3\sin t, 4t \rangle$

$\vec{r}\,' = \langle -3\sin t,\ 3\cos t,\ 4 \rangle$ & $\vec{r}\,'' = \langle -3\cos t,\ -3\sin t,\ 0 \rangle$

$|\vec{r}\,'| = 5$

$\Rightarrow \vec{T} = \left\langle -\frac{3}{5}\sin t,\ \frac{3}{5}\cos t,\ \frac{4}{5} \right\rangle$

$\vec{T}\,' = \left\langle -\frac{3}{5}\cos t,\ -\frac{3}{5}\sin t,\ 0 \right\rangle$

$|T'| = \frac{3}{5}$

$\Rightarrow \vec{N} = \langle -\cos t,\ -\sin t,\ 0 \rangle$

$$\Rightarrow \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\frac{3}{5}s & \frac{3}{5}c & \frac{4}{5} \\ -c & -s & 0 \end{vmatrix} = \left\langle \frac{4}{5}\cdot s,\ \frac{-4}{5}c,\ \frac{3}{5} \right\rangle$$

and

$$\vec{r}\,' \times \vec{r}\,'' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3s & 3c & 4 \\ -3c & -3s & 0 \end{vmatrix} = \langle +12s,\ -12c,\ 9 \rangle$$

w/ magnitude 15

$\Rightarrow \kappa = \frac{15}{5^3} = \frac{3}{25}$

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