12.5: Equations of Lines and planes

LINES

Recall from Friday (animation), that a line thru the point given by position vector $\vec{r}_0$ and in the direction of $\vec{v}$, can be expressed by the vector equation $\vec{r} = \vec{r}_0 + t\cdot\vec{v}$ (t a parameter).

If $\vec{r} = \langle x, y, z\rangle$
$\vec{r}_0 = \langle x_0, y_0, z_0\rangle$
$v = \langle a, b, c\rangle$, then

$$\langle x, y, z\rangle = \langle x_0 + ta, y_0 + tb, z_0 + tc\rangle$$

and we have the parametric equations of a line L thru (x_0, y_0, z_0) parallel to $\vec{v}$

$$x = x_0 + ta, \quad y = y_0 + tb, \quad z = z_0 + tc, \quad t \in \mathbb{R}$$

<u>Ex1</u>: Find the vector equation of the line thru $(1,2,3)$ parallel to $\langle 4,5,6\rangle$

Eliminating the parameter gives the symmetric equations

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c} \quad \text{when } a,b,c \neq 0.$$

If (for example) $a=0$, $x = x_0$, $\frac{y-y_0}{b} = \frac{z-z_0}{c}$
which is a line on the plane $x = x_0$.

Ex2: Find the symmetric equations of the line in (ex1) & the pt where it intersects the xy-plane. ← (when $z=0$).

To describe the line from $\vec{r}_0$ to $\vec{r}_1$, we have

$\vec{r} = (1-t)\vec{r}_0 + t\vec{r}_1$, $0 \le t \le 1$. (don't memorize).

PLANES

Planes can be determined by a point and a normal vector (orthogonal). why?

If $\vec{r}$ and $\vec{r}_0$ are position vectors of points on the plane w/ normal $\vec{n}$, then $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$.

This can be written as $\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{r}_0$ (vector equation of the plane).

If $\vec{n} = \langle a, b, c \rangle$

$\vec{r} = \langle x, y, z \rangle$

and $\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$, then

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = \langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle$$

$$= a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Ex3: Find the equation of the plane thru $(1,2,3)$ w/ normal $\vec{n} = \langle 4, 5, 6 \rangle$

Question: How would I find the equation of the line thru 3 points that are not colinear?

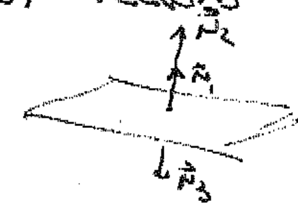
Defn. We define the angle between planes to be the angle between normals.

Ex4: Find the parametric equations for the line of intersection of the planes $z = x+y$ & $2x-5y-z=1$

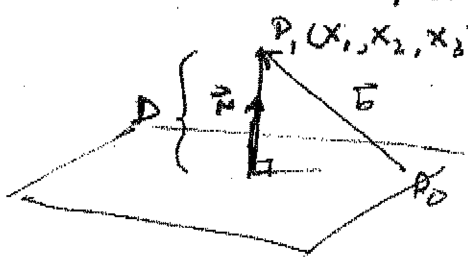
a) use $\vec{n}_1 \times \vec{n}_2$ to find $\vec{v}$ // to the line. L.

b) Let $z=0$ to find a point on L.

NOTE: There are an infinite number of vectors Normal to a plane at a point



Find the distance from a plane to a point.



We need the length of $\vec{D}$. It is not enough to know the direction. $\vec{n} = \langle a, b, c \rangle$.

suppose $P_0(x_0, y_0, z_0)$ is a point on $ax+by+cz+d=0$.

$\vec{b} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$. Now $D = |\text{comp}_{\vec{n}} \vec{b}|$

$$= \frac{|\vec{n} \cdot \vec{b}|}{|\vec{n}|}$$

$$\frac{|\vec{n}\cdot\vec{b}|}{|\vec{n}|} = \frac{|a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0)|}{|\vec{n}|}$$

circled terms sum to zero since on the plane.

$$= \frac{|ax_1 - ax_0 + by_1 - by_0 + cz_1 - cz_0 + d - d|}{|\vec{n}|} = \frac{|ax_1 + by_1 + cz_1 + d|}{|\vec{n}|}.$$