3.3: Gauss-Jordan Elimination

Three ops.

stand up / sit down exercise
to teach order

3.3
2/5

ex: $\begin{cases} 2x - y = 5 \\ x + 3y = 6 \end{cases}$

$\Rightarrow \left[\begin{array}{cc|c} 2 & -1 & 5 \\ 1 & 3 & 6 \end{array}\right] \quad R_1 \leftrightarrow R_2$

$\Rightarrow \left[\begin{array}{cc|c} 1 & 3 & 6 \\ 2 & -1 & 5 \end{array}\right] \quad R_2 - 2R_1 \to R_2$

$\Rightarrow \left[\begin{array}{cc|c} 1 & 3 & 6 \\ 0 & -7 & -7 \end{array}\right] \quad -\frac{1}{7} R_2 \to R_2$

$\Rightarrow \left[\begin{array}{cc|c} 1 & 3 & 6 \\ 0 & 1 & 1 \end{array}\right] \quad R_1 - 3R_2 \to R_1$

$\Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \end{array}\right]$

$x = 3$ and $y = 1$.

3.3
3/5

Ex: $\begin{cases} x + 2y - z = 3 \\ 2x + 5y - 2z = 7 \\ -x + y + 5z = -12 \end{cases}$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 5 & -2 & 7 \\ -1 & 1 & 5 & -12 \end{array}\right] \quad R_2 - 2R_1 \to R_2$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ -1 & 1 & 5 & -12 \end{array}\right] \quad R_3 + 1R_1 \to R_3$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 3 & 4 & -9 \end{array}\right] \quad R_3 - 3R_2 \to R_3$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 4 & -12 \end{array}\right] \quad \frac{1}{4} R_3 \to R_3$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right] \quad R_1 + 1R_3 \to R_1$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right] \quad R_1 - 2R_2 \to R_1$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

$x = -2$
$y = 1$
$z = -3$

3.3
4/5

ex:
$$\begin{cases} 2x + 3y + 4z = +2 \\ x + 2y + 2z = 1 \\ x + y + 2z = 2 \end{cases}$$

$$\rightarrow \left[\begin{array}{ccc|c} 2 & 3 & 4 & 2 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 2 & 2 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 2 & 3 & 4 & 2 \\ 1 & 1 & 2 & 2 \end{array}\right] \quad R_2 - 2R_1 \rightarrow R_2$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 1 & 2 & 2 \end{array}\right] \quad R_3 - R_1 \rightarrow R_3$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] \quad -R_2 \rightarrow R_2$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] \quad R_3 + R_2 \rightarrow R_3$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \quad 0 \neq 1$$

no soln.

ex:

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + 4z = 2 \Rightarrow x = 2 - 4z$

&

$y + z = 3 \Rightarrow y = 3 - z$

infinite # of soln.