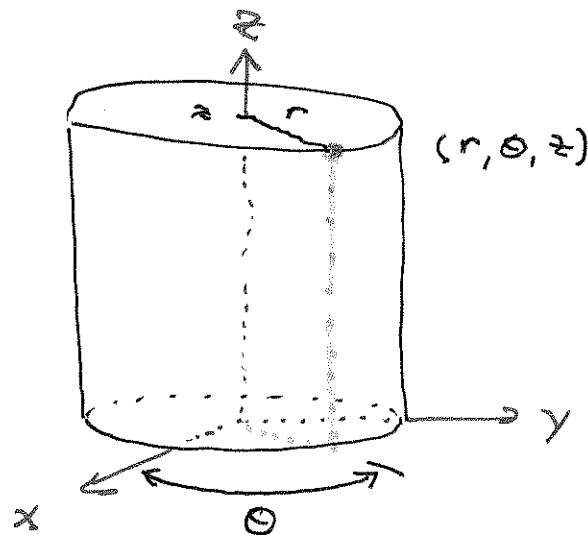


## 15.7: Triple Integrals in Cylindrical Coords

15.8  
13

cylindrical  
coordinates

(sweet mathematics  
visualization).



### Differentials

single integral :

$$dx \quad \xrightarrow{dx}$$

double integral :

$$dA \quad \begin{array}{c} dy \\ | \\ dx \end{array} \quad dA = dx \cdot dy$$

$$dA = r dr d\theta$$

A diagram showing a sector of a circle in the  $xy$ -plane, bounded by  $r$  and  $r + dr$  from the origin, and subtended by an angle  $d\theta$ .

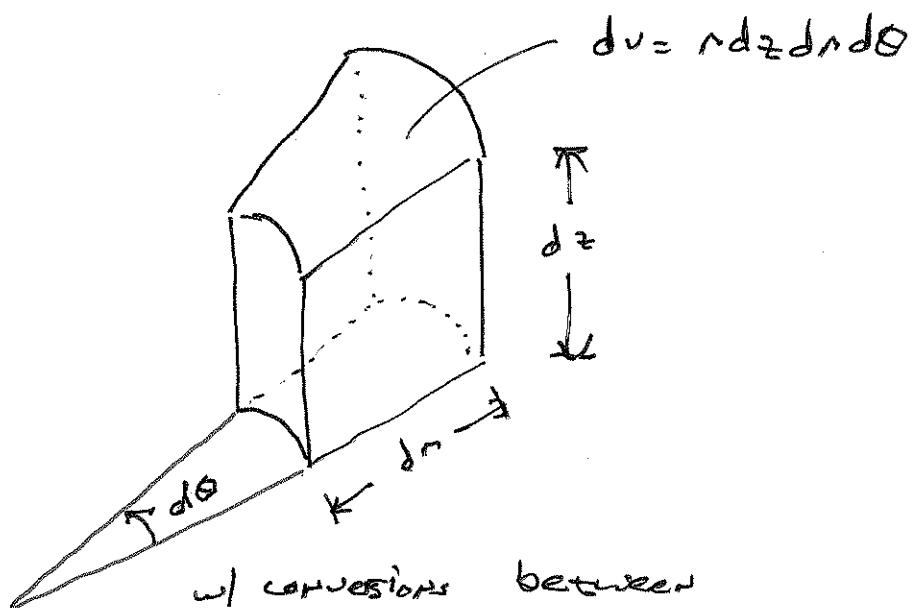
triple integrals :

$$dv$$

$$dv = dx dy dz$$

A diagram of a rectangular prism in a 3D Cartesian coordinate system. The vertical axis is  $z$ , the horizontal axis pointing out of the page is  $y$ , and the axis pointing into the page is  $x$ . The volume element is labeled  $dz$ ,  $dy$ , and  $dx$ .

15.8  
2/3



w/ conversions between  
coord systems of

$$x = r \cos \theta ; \quad y = r \sin \theta$$

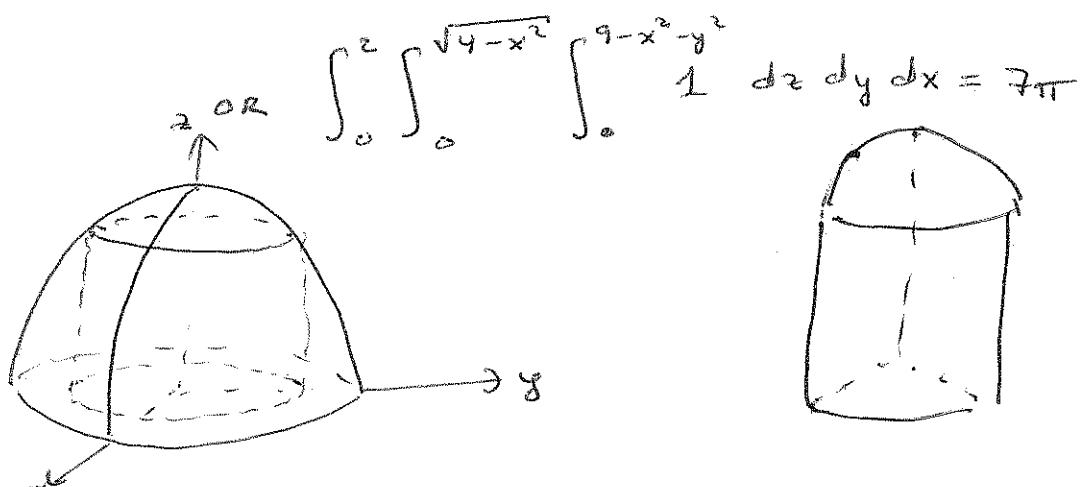
$$x^2 + y^2 = r^2 ; \quad \frac{y}{x} = \tan \theta$$

Ex 1: (A) Sketch the solid whose volume is given

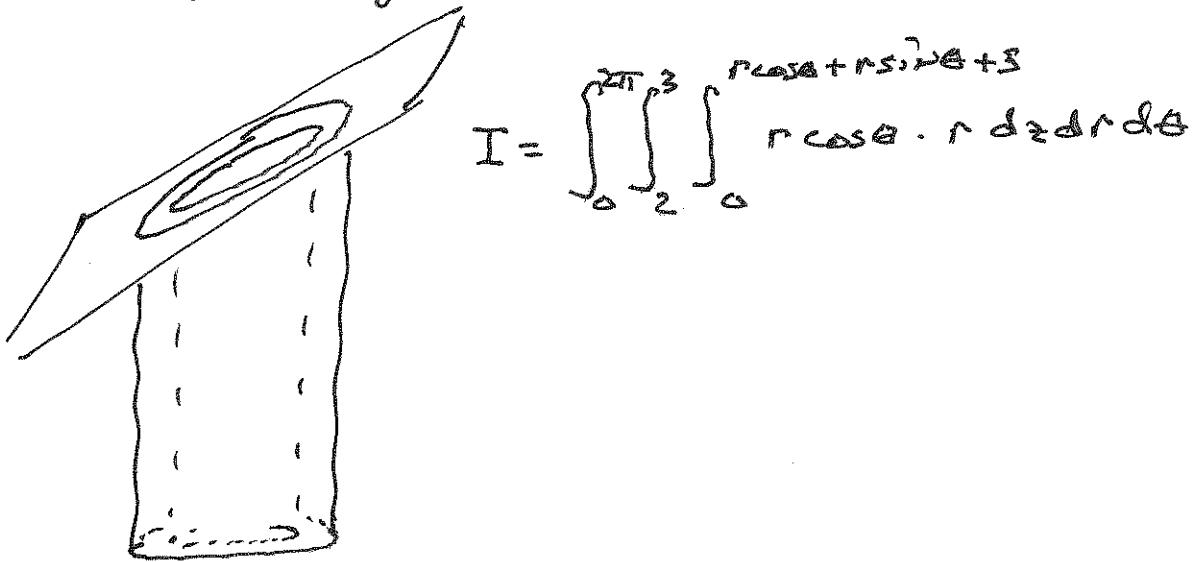
$$\text{by } \int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} 1 \cdot r \, dz \, dr \, d\theta = 7\pi$$

(B) Express the volume using cartesian coords

(C) calculate the volume whichever  
way seems easier.

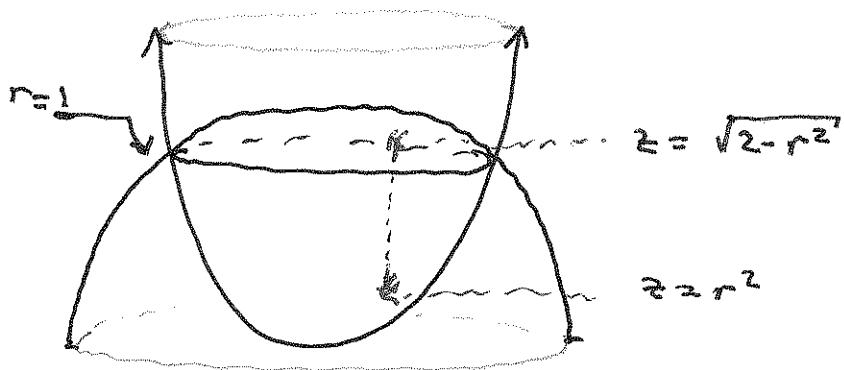


ex2: Set-up an iterated integral to find  $I = \iiint_E x dV$  where  $E$  is enclosed by  $z=0$ ,  $z=x+y+5$ ,  $x^2+y^2=4$ , and  $x^2+y^2=9$



$$I = \int_0^{2\pi} \int_0^3 \int_0^{r \cos \theta + r \sin \theta + 5} r \cos \theta \cdot r dz dr d\theta$$

ex3: Find the volume of the solid that lies between the paraboloid  $z = x^2 + y^2$  and the sphere  $x^2 + y^2 + z^2 = 2$



$$\text{Volume } V = 4 \int_{\frac{\pi}{2}}^{\pi} \int_{0}^1 \int_{r^2}^{\sqrt{2-r^2}} 1 \cdot r dz dr d\theta$$