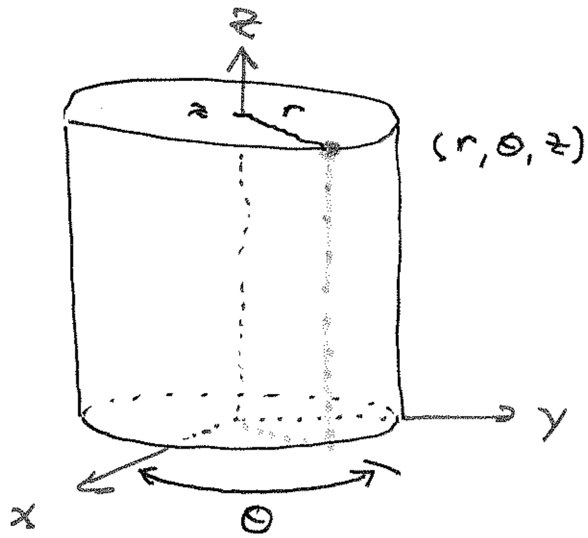
15.7: Triple Integrals in Cylindrical Coords

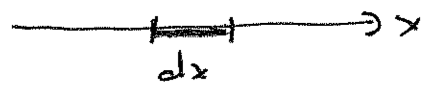
cylindrical coordinates

(sweet mathematica visualization).

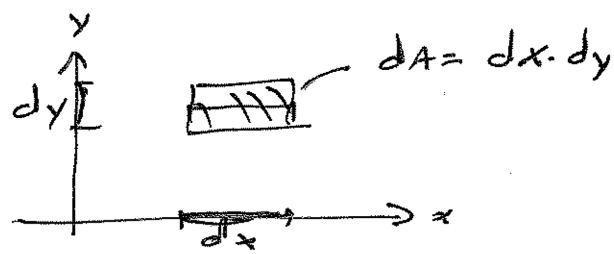


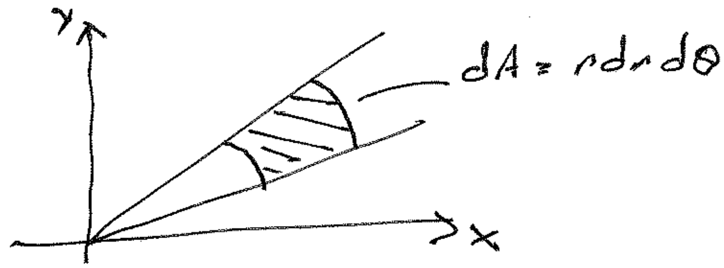
Differentials

single integral : dx

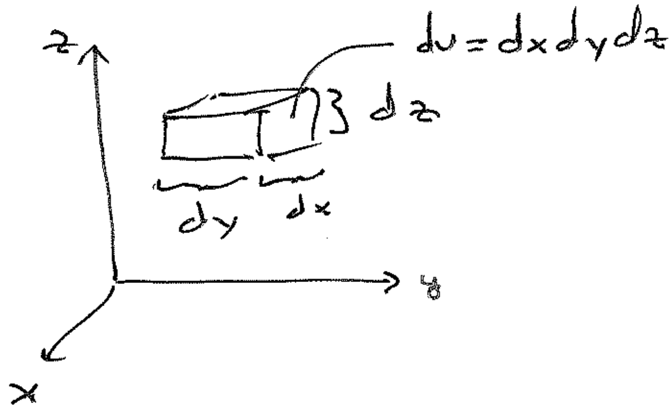


double integral : dA

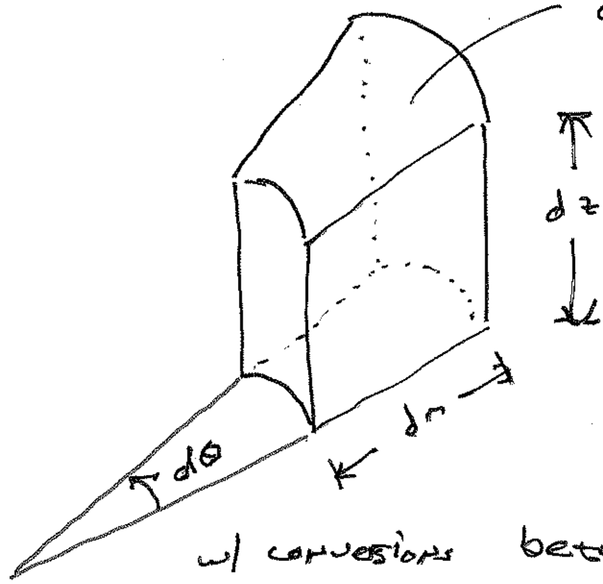




triple integrals : dV



$dv = r\,dz\,dr\,d\theta$



w/ conversions between coord systems of

$x = r\cos\theta \;; \quad y = r\sin\theta$

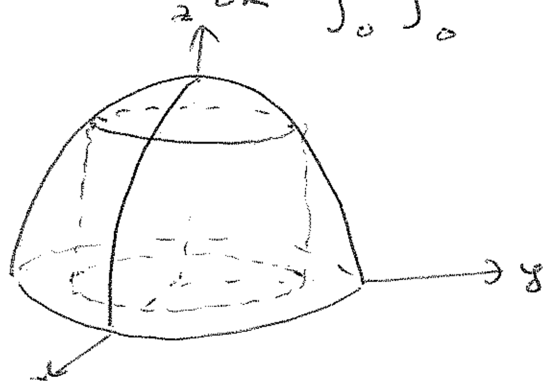
$x^2 + y^2 = r^2 \;; \quad \dfrac{y}{x} = \tan\theta$

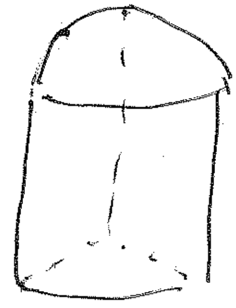
<u>Ex1</u>: (A) Sketch the solid whose volume is given by $\displaystyle\int_0^{\pi/2}\int_0^{2}\int_0^{9-r^2} 1 \cdot r\,dz\,dr\,d\theta = 7\pi$

(B) Express the volume using cartesian coords

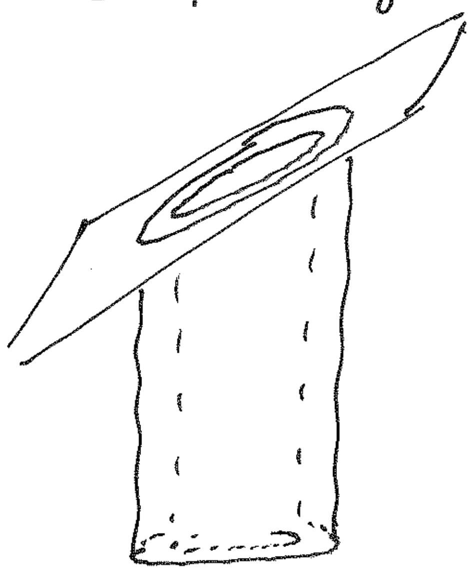
(C) calculate the volume whichever way seems easier.

OR $\displaystyle\int_0^{2}\int_0^{\sqrt{4-x^2}}\int_0^{9-x^2-y^2} 1 \; dz\,dy\,dx = 7\pi$



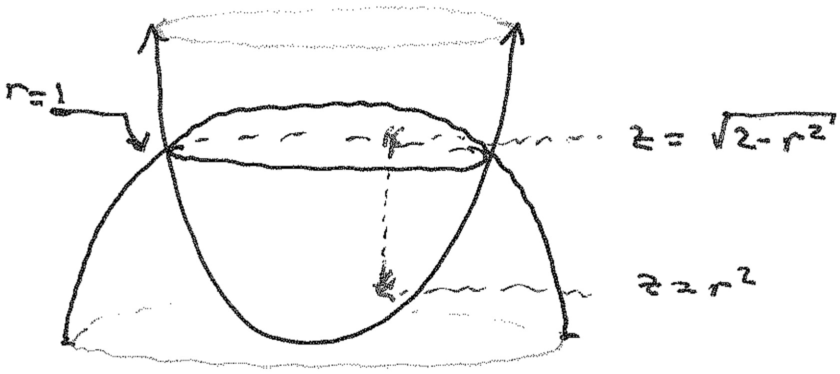


ex2: Set-up an iterated integral to find $I = \iiint_E x\,dV$ where E is enclosed by $z=0$, $z = x+y+5$, $x^2+y^2=4$, and $x^2+y^2=9$



$$I = \int_0^{2\pi}\int_2^3\int_0^{r\cos\theta + r\sin\theta + 5} r\cos\theta \cdot r\,dz\,dr\,d\theta$$

ex3: Find the volume of the solid that lies between the paraboloid $z = x^2+y^2$ and the sphere $x^2+y^2+z^2 = 2$



Volume $V = 4\int_0^{\frac{\pi}{2}}\int_0^1\int_{r^2}^{\sqrt{2-r^2}} 1 \; r\,dz\,dr\,d\theta$