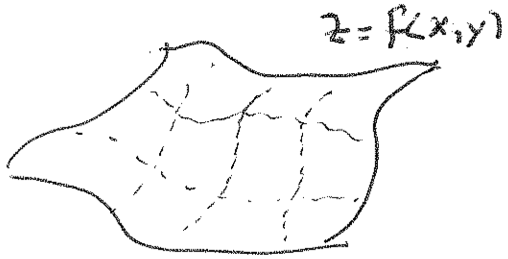
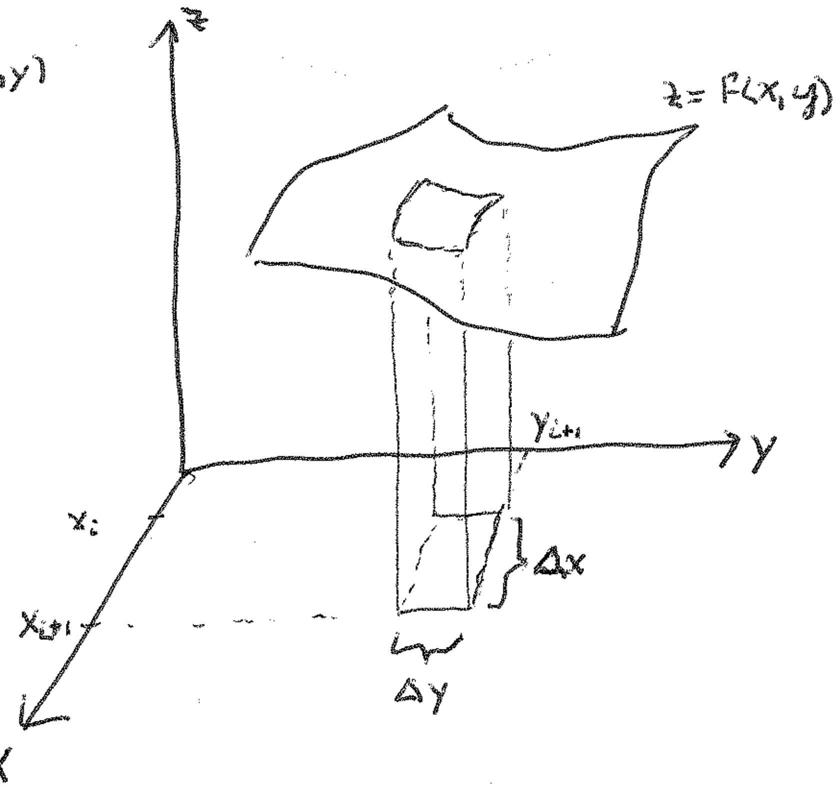
15.6: Surface Area

Find the SA of $z = f(x,y)$ over D.

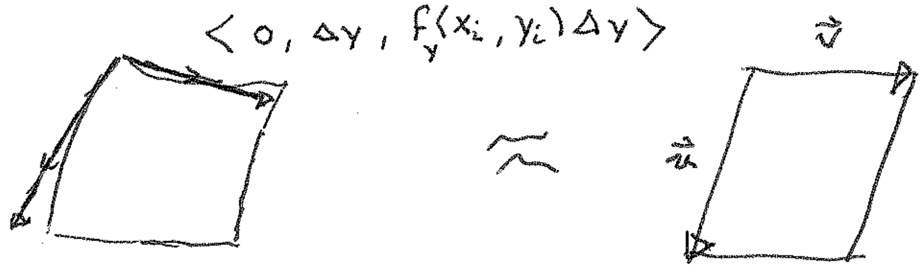


Find the area of each patch



$\vec{v}$

$\langle 0, \Delta y, f_y(x_i, y_i)\Delta y \rangle$



$\vec{u} = \langle \Delta x, 0, f_x(x_i, y_i)\Delta x \rangle$

The area of the patch is approx. the area of the parallelogram $= |\vec{u} \times \vec{v}|$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \Delta x & 0 & f_x \Delta x \\ 0 & \Delta y & f_y \Delta y \end{vmatrix}$$

$$= \langle -f_x \Delta x \Delta y, \ -f_y \Delta x \Delta y, \ \Delta x \Delta y \rangle$$

$$\Rightarrow |\vec{u} \times \vec{v}| = \sqrt{(f_x^2 + f_y^2 + 1)} \, \Delta x \Delta y$$

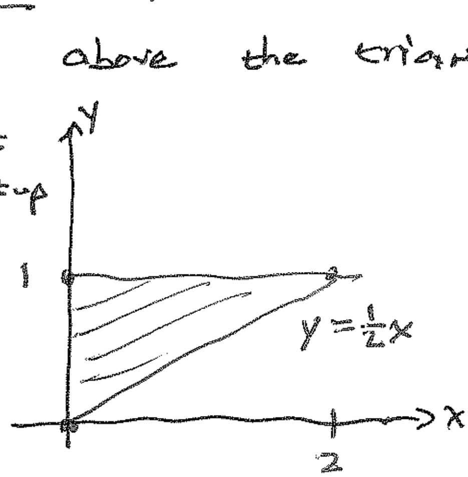
This is the approx area of one patch. To find the exact area, let $\Delta x, \Delta y \to 0$ and add up the areas.

The area of the surface S w/eqt $z = f(x,y)$ w/ $(x,y) \in D$, where f_x and f_y are cont. is

$$A(S) = \iint_D \sqrt{[f_x(x,y)]^2 + [f_y(x,y)]^2 + 1} \, dA$$

Ex 1: Find the SA of $z = 1 + 3x + 2y^2$ that lies above the triangle w/ vertices $(0,0)$, $(0,1)$, $(2,1)$

NOTE: If rushed for time, just do the setup



$z_x = 3, \ z_y = 4y$

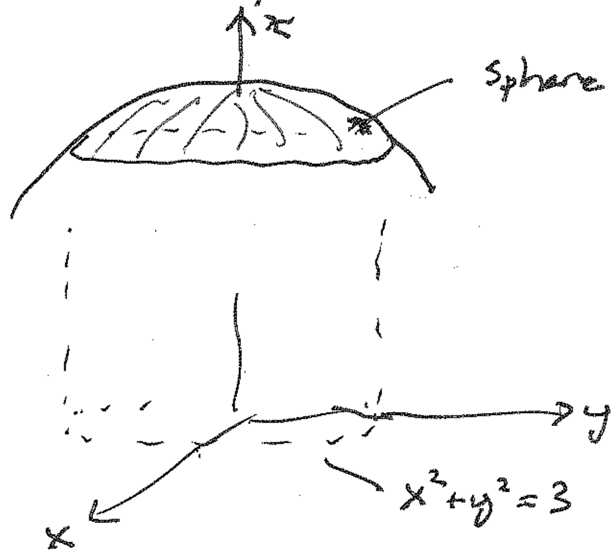
and $\sqrt{(z_x)^2 + (z_y)^2 + 1} = \sqrt{9 + 16y^2 + 1}$

$$SA = \int_0^1 \int_0^{2y} \sqrt{10 + 16y^2} \, dx \, dy$$

$$= \int_0^1 2y \sqrt{10 + 16y^2} \, dy$$

$$= \left[\frac{1}{24} (10 + 16y^2)^{3/2} \right]_0^1 \quad \frac{1}{24} \left(26^{3/2} - 10^{3/2} \right)$$

ex2: set up an iterated integral to find the part of $x^2+y^2+z^2=4$ above $z=1$.



$$z=\sqrt{4-x^2-y^2}$$

$$z_x = -\frac{x}{\sqrt{4-x^2-y^2}}$$

$$z_y = -\frac{y}{\sqrt{4-x^2-y^2}}$$

set up the integral in polar.

$$SA = \int_0^{2\pi}\int_0^{\sqrt{3}} \sqrt{\frac{r^2\cos^2\theta}{4-r^2}+\frac{r^2\sin^2\theta}{4-r^2}+1}\; r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt{3}} r\sqrt{\frac{4}{4-r^2}}\,dr\,d\theta$$