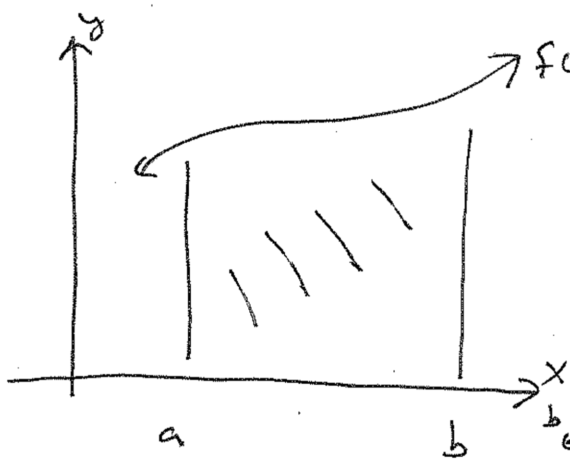
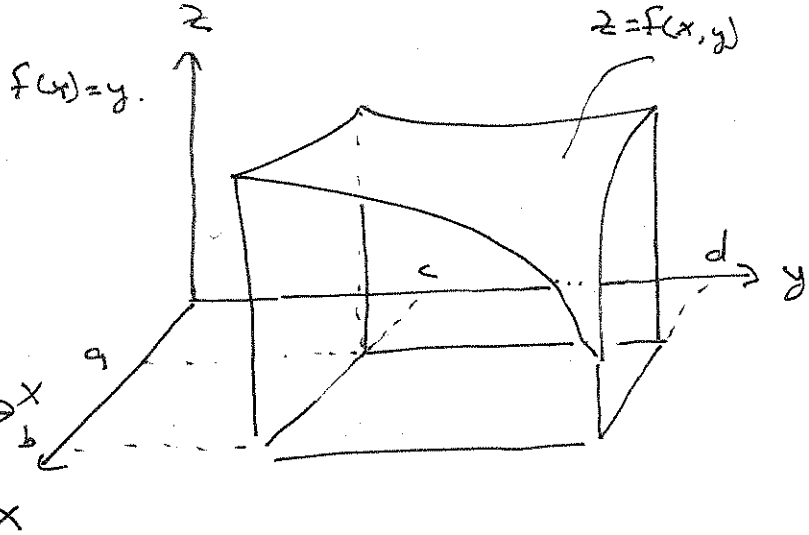
15.1: Double Integrals over rectangles

This section is analogous to 5.2 which introduced the definite integral.





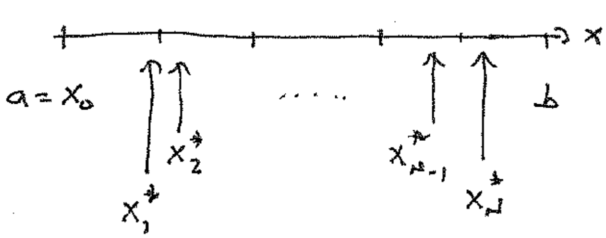
Find the Area

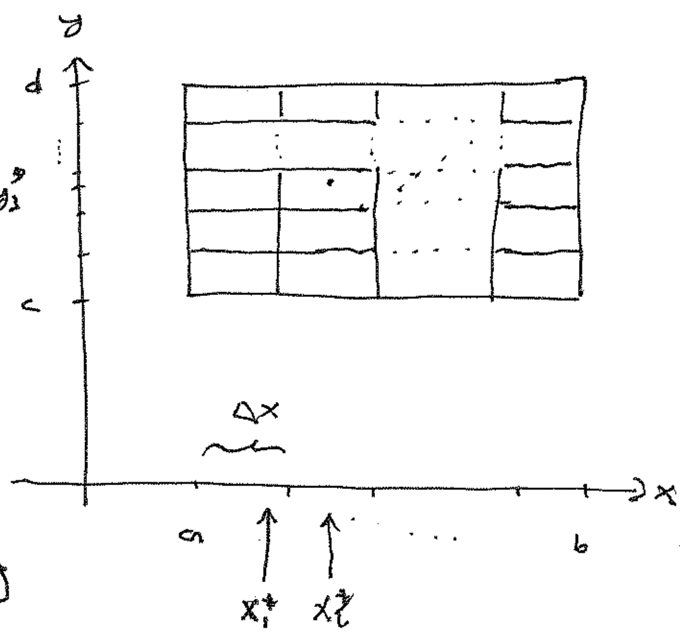
$f \geq 0$

Find the volume

$f \geq 0$

we can break up our domains





The ij^{th} subrectangle

$R_{ij} = \{(x,y) \mid x_{i-1} \leq x \leq x_i$ and $y_{j-1} \leq y \leq y_j\}$

Assuming that our fcts are well chosen & the region of integration is partitioned correctly

$$A = \int_a^b f(x)\,dx = \lim_{N\to\infty} \sum_{i=1}^{N} f(x_i^*)\Delta x$$

$$V = \int_a^b\int_c^d f(x, y)\,dA = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A.$$

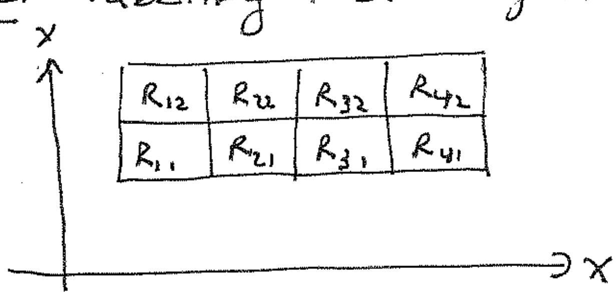
Then we approx the area/volume w/ rectangles/boxes (prisms).

Evaluating the limit (provided it exists) increases the number of boxes/rectangles $\to \infty$ and in the limit provides us w/the exact area/volume.

ex1: reading double sums.

expand $\displaystyle\sum_{i=1}^{3}\sum_{j=1}^{2} A_{ij}\Delta A$

ex2: labeling rect. regions.



1st index $\to$ x

2nd index $\to$ y.

ex3: consider $z = x + 2y^2$ over $[0,4] \times [1,4]$

(a) est. the vol. under the surface w/ Riemann sums. Let $m = 2$ & $n = 3$. Use sample pts in the lower left.

(b) same as above ... but mid pts

(c) use the midpt rule to est. the ave. val. of the fct.

$$f_{ave} = \frac{1}{A(R)} \iint_R f(x,y)\, dA$$

ex4: Find $\int_0^{\pi}\int_0^{\pi} \sin(x+y)\, dx\, dy$. by first identifying the volume as a solid.