Review/Introduction to Matrices. for ch8.

matrix multiplication.

Write systems as a matrix product

Determinants

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & -2 & 5 \\ 1 & 0 & 7 \\ 0 & 0 & 2 \end{bmatrix}$$

Eigenvectors and Eigenvalues

$$A\vec{v} = \lambda\vec{v}$$

$$\Rightarrow A\vec{v} - \lambda\vec{v} = \vec{0}$$

$$\Rightarrow (A - \lambda I)\vec{v} = \vec{0}$$

<u>1st</u>: Find eigenvalues by solving $\det(A - \lambda I) = 0$.

<u>2nd</u>: For each eigenvalue, find the corresponding eigenvector.

8.1 : Prelim Theory – Linear systems

1st order System

$$\frac{dx}{dt} = a_{11}(t)x + a_{12}(t)y + a_{13}(t)z$$

$$\frac{dy}{dt} = a_{23}(t)x + a_{22}(t)y + a_{23}(t)z$$

$$\frac{dz}{dt} = a_{31}(t)x + a_{32}(t)y + a_{33}(t)z$$

homogeneous example w/ 3 variables.

$$\hookrightarrow X' = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} X$$

$$\frac{dx}{dt} = a_{11}(t)x + a_{12}(t)y + f_1(t)$$

$$\frac{dy}{dt} = a_{21}(t)x + a_{22}(t)y + f_2(t)$$

Non-homogeneous w/2 variables

$$\hookrightarrow X' = \underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{22} & a_{22} \end{bmatrix}}_{A} X + \underbrace{\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}}_{F}$$

Def: A solution vector on an interval I is of the form : $X = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$ whose entries are differentiable fcts satisfying $X' = AX + F$

ex1: verify $X = \begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{-3t/2}$ is a soln to

$$X' = \begin{bmatrix} -1 & 1/4 \\ 1 & -1 \end{bmatrix} X.$$

As before, we have IVPs, then w/ nice A and F each IVP has a unique soln.

Superposition still works. That is, if $\mathbf{X}_1, \dots, \mathbf{X}_k$ are solns to $\mathbf{X}' = A\mathbf{X}$, then $\mathbf{X} = c_1\mathbf{X}_1 + \dots + c_k\mathbf{X}_k$ is a soln. for arbitrary constants $c_1, \dots, c_k$

<u>Dfn</u>: The set of solns $\mathbf{X}_1, \dots, \mathbf{X}_k$ is <u>linearly dependent</u> if $\exists\, c_1, \dots, c_k$, not all zero, s.t. $c_1\mathbf{X}_1 + \dots + c_k\mathbf{X}_k = \vec{0}$ for all t on an interval. If not L.D., the vectors are linearly independent.

In a sys. of n eqns, $\mathbf{X}_1, \dots, \mathbf{X}_n$ are L.I. iff

$$\begin{vmatrix} | & & | \\ \mathbf{X}_1 & \cdots & \mathbf{X}_n \\ | & & | \end{vmatrix} \neq 0 \quad \leftarrow \text{Wronskian} \neq 0.$$

If the set is L.I., we call it a fundamental set.

<u>ex2</u>: Is this a fundamental set?

(a) $\vec{x}_1 = e^{2t}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\vec{x}_2 = e^{2t}\begin{bmatrix} -2 \\ 4 \end{bmatrix}$

(b) $\vec{x}_1 = e^{-t}\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\vec{x}_2 = e^{4t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The general, complementary, and particular solns are as before.

If $\bar{X}_1, \dots, \bar{X}_n$ are a fundamental set of solns for $\bar{X}' = A\bar{X}$

Then $\bar{X}_c = c_1\bar{X}_1 + \dots + c_n\bar{X}_n$ is the complementary soln.

And if $\bar{X}_p$ is a soln to $\bar{X}' = A\bar{X} + F$, then

The gen. soln. is $\bar{X} = \bar{X}_c + \bar{X}_p$.