4.4: The Superposition Principle and undetermined coefficients.

The method of good guessing. (kinda like partial fractions)

ex1: $y'' + 3y' + 2y = 3x$; $y_p = Ax + B$

$$\downarrow$$

$$\frac{3}{2}x - \frac{9}{4}$$

ex2: $y'' + 3y' + 2y = 10e^{3x}$; $y_p = Ae^{3x}$

$$\downarrow$$

$$\frac{1}{2}e^{3x}$$

ex3: $y'' + 3y' + 2y = 3x + 10e^{3x}$

ex4: $y'' + y' = 5$ (no constant... try linear)

ex5: $y'' - 3y' + 9y = e^{3t}$ (no e^{3t}, te^{3t}, ... try t^2e^{3t})

4.4: Undetermined Coefficients – Superposition Approach

Form: $a_n(x)y^{(n)} + \ldots + a_1(x)y' + a_0(x)y = g(x)$

This method works if $g(x)$ is a linear combination of functions of the following types:

1.) $P(x) = a_n x^n + \ldots + a_1 x + a_0$ (polynomials)

2.) $P(x)e^{\alpha x}$

3.) $P(x)e^{\alpha x}\sin(\beta x)$ and $P(x)e^{\alpha x}\cos(\beta x)$

Note: We say a function *y* is a linear combination of f_1 ...f_n if $y = c_1 f_1 + \dots + c_n f_n$ for constants $c_1, \ldots, c_n$.

TABLE 4.4.1 Trial Particular Solutions

$g(x)$	Form of y_p
1. 1 (any constant)	A
2. $5x + 7$	$Ax + B$
3. $3x^2 - 2$	$Ax^2 + Bx + C$
4. $x^3 - x + 1$	$Ax^3 + Bx^2 + Cx + E$
5. $\sin 4x$	$A\cos 4x + B\sin 4x$
6. $\cos 4x$	$A\cos 4x + B\sin 4x$
7. e^{5x}	Ae^{5x}
8. $(9x - 2)e^{5x}$	$(Ax + B)e^{5x}$
9. x^2e^{5x}	$(Ax^2 + Bx + C)e^{5x}$
10. $e^{3x}\sin 4x$	$Ae^{3x}\cos 4x + Be^{3x}\sin 4x$
11. $5x^2\sin 4x$	$(Ax^2 + Bx + C)\cos 4x + (Ex^2 + Fx + G)\sin 4x$
12. $xe^{3x}\cos 4x$	$(Ax + B)e^{3x}\cos 4x + (Cx + E)e^{3x}\sin 4x$