2.2: Separable Equations

Ex 1: $\frac{dy}{dx} = \frac{y}{x}$

Ex 2: $(x^2+1)y' = xy$

Ex 3: $\frac{du}{dt} = 2 + 2u + t + tu$

Ex 4: $\frac{dy}{dx} = y^2+1, \quad y(1)=0$

Ex 5: $\frac{dP}{dt} = \sqrt{Pt}, \quad P(1)=2$

Ex 6: $xy' + y = x^2, \quad y(1) = -1$

IVPs

Ex 7: A small country has \$10 billion in paper currency in circulation. Each day \$50 million comes into banks. The government decides to replace old bills w/new when the old comes into banks. How long until the new bills account for 90% of the currency in circulation?

Let $x(t)$ = amount of new money in circ.

Ans. 460.5 days.

$\hookrightarrow$ $X(0) = 0$

$\dfrac{X}{50\text{ bil}}$ = proportion of new currency in the system.

$1 - \dfrac{X}{50\text{ bil}}$ = prop. of old currency in the system.

$\dfrac{dx}{dt} = \begin{pmatrix}\text{rate new} \\ \$ \text{ enters}\end{pmatrix} - \begin{pmatrix}\text{rate new} \\ \$ \text{ exits}\end{pmatrix}$

$\Rightarrow \dfrac{dx}{dt} = 50\text{ mil.}\left(1 - \dfrac{x}{50\text{ bil}}\right) - (0)$

$= \dfrac{1}{1000}(50\text{ bil} - x)$

$\Rightarrow \dfrac{dx}{50\text{ bil} - x} = \dfrac{1}{1000}\,dt$

$\Rightarrow -\ln(50\text{ bil} - x) = \dfrac{1}{1000}t + c$

$\Rightarrow 50\text{ bil} - x = ke^{-1/1000\,t}$

$\Rightarrow x = 50\text{ bil} - ke^{-1/1000\,t}$

$\Rightarrow \quad = 50\text{ bil}\left(1 - e^{-1/1000\,t}\right)$

Faster soln

$c = -\ln(50\text{ bil})$

$\Rightarrow t = 1000\ln\left(\dfrac{50\text{ bil}}{50\text{ bil} - x}\right)$

and when $x = 45\text{ bil}$

$\Rightarrow t = 1000\ln(10)$

$= 2303$ days