1.2 : IVPs

general vs. particular solns.

ex1: consider $y' = 1 + y^2$

verify $y = \tan(x + c)$ is a soln.

solve the IVP given $y(0) = 0$

ex2: consider $y' = y^2$

verify $y = \frac{-1}{x + c}$ is a soln.

solve the IVP given (a) $y(0) = 1$

(b) $y(0) = -1$

interval of defn.

Domain of a fcn vs. domain of the soln

Justification: DE's involve derivatives. Since differentiable is a stronger condition than continuity, the soln must be cont.

ex1, 2 rev: find the intervals of definition for the examples above.

vocab: implicit vs. explicit soln.

$x^2 + y^2 = 4$ vs. $y = -\sqrt{4 - x^2}$

ex3: verify the implicit solution

$\ln\left(\frac{2x-1}{x-1}\right) = t$ for $\frac{dx}{dt} = (x-1)(1-2x)$

use implicit diff

$$\frac{\cancel{x-1}}{2x-1} \cdot \frac{2\frac{dx}{dt}(x-1) - 1\frac{dx}{dt}(2x-1)}{(x-1)^{\cancel{2}}} = 1$$

$$\Rightarrow \frac{dx}{dt} \cdot \frac{2(x-1)-(2x-1)}{(2x-1)(x-1)} = 1$$

$$\Rightarrow \frac{dx}{dt} = -(2x-1)(\cancel{2}x-1)$$

$$= (x-1)(1-2x) \checkmark$$

The trivial soln $y=0$ (see ex1 & ex2)

Math concept: existence and uniqueness

(see ex2).

Thm: existence of a unique soln.

consider $\frac{dy}{dx} = f(x,y)$. $y(x_0) = y_0$. Let R be ~~the~~ a region $[a,b] \times [c,d]$ containing (x_0, y_0) in the xy-plane. If $f(x,y)$ and $\frac{\partial f}{\partial y}$ are cont. on R then $\exists$ an open interval I_0 containing x_0 and a unique fcn $y(x)$ defined on I_0 that is a soln to the IVP.

different sets: suppose $y(x)$ is a solution ~~to~~ an IVP. The following may be different:

(1) the domain of $y(x)$

(2) the interval over which $y(x)$ is defined (or exists)

(3) The interval where the solution is unique and exists.