

1.1: Definitions and terminology

DEF: An eqn. containing the derivatives of one or more unknown fcts (or dependent variables) w.r.t one or more ind. vars. is said to be a diff. eqn (DE).

vocab:

ODE vs. PDE.

order

linear vs. non-linear ODEs.

examples of ODEs

ex 1: (a) $\frac{dy}{dx} - 2y = \sin x$

(b) $y'' - 4y' + 6y = 0$

(c) $\frac{dy}{dz} + \frac{dx}{dz} = e^y + \sin(x)$

basic idea of partial derivatives

ex 2: Find partials

(a) $f(x, y) = x \sin y$

(b) $f(x, y, z) = x^2 y^3 z^4$

ex 3: examples of PDEs

$$(a) \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y} = xy$$

$$(b) \frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial y^2} - 3x \frac{\partial v}{\partial x}$$

$$(c) \frac{\partial f}{\partial x} = - \frac{\partial f}{\partial y}$$

order = order of the highest deriv.

linear eqts

$$2x = 1$$

$$2x + 3y = 4$$

$$4x + 3y + 2z = 5$$

linear DE.

$$y' + y = 1$$

$$a(x)y' + b(x)y = 1$$

$$a(x)y'' + b(x)y' + c(x)y = 0$$

NOE: $y \cdot y' + y = 1$

$$a(x)y' + b(x)\sin(y) = 1$$

$$y'' + y^2 = c$$

verify solns:

ex 4: $y'' + 4y = 0, y = \cos 2x$

ex 5: $y' + 2y = 0, y = 3e^{-2x}$

IvP: $y' = y + 1, y = Ce^x - 1, y(0) = 5$

$y' = x - y, y = Ce^{-x} + x - 1, y(0) = 10$

Sometimes first order DEs are written as

$$M(x, y) dx + N(x, y) dy = 0$$

ex 6: $(2xy^2 - 3) dx + (\cos x + x \cos y) dy$

$$x \frac{dy}{dx} = 2xe^x - y$$

ex 7: Are these linear?

$$(x^2 + \frac{2y}{x}) dx = (3 - 4x^2) dy$$

$$y dx = (y - x y^2) dy$$