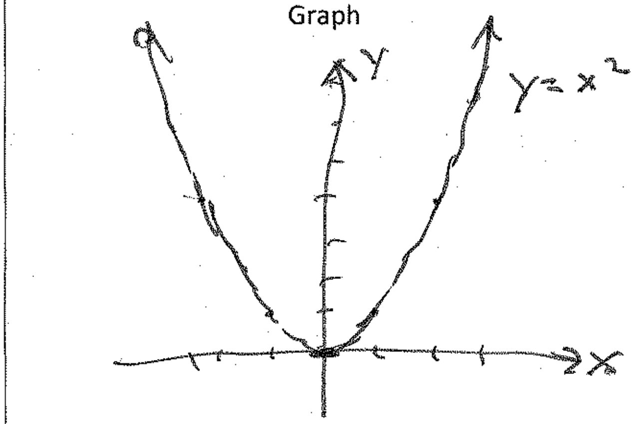
Graphing Quadratics (8.6 & 7) **Math 098**

Graphing quadratic functions requires a strong understanding of the "toolkit" function $f(x) = x^2$

Table

x	x^2
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Graph



With that toolkit knowledge, we can graph the transformed "toolkit" quadratic $f(x) = a(x-h)^2 + k$

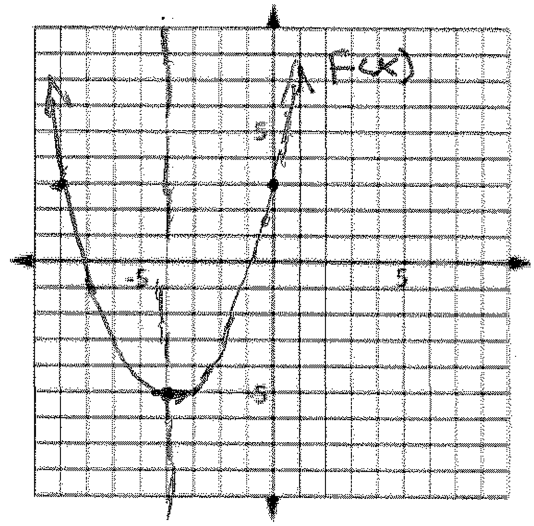
Let's explore each of the parameters: *a, h,* and *k.*

- $f(x) = \boldsymbol{a}(x-h)^2 + k$
  - If $a > 0$, the quadratic is smiley or, more precisely, concave up.
  - If $a < 0$, the quadratic is frowny or, more precisely, concave down
- $f(x) = a\left(x - \boldsymbol{h}\right)^2 + k$
  - Shift the quadratic left or right in the opposite direction of the sign you see.
- $f(x) = a(x-h)^2 + \boldsymbol{k}$
  - Shift the quadratic up or down in the same direction as the sign you see.

Note: A more rigorous development of these concepts (including the reasons why they work) can be found in section 8.6 of the text.

Example 1: Graph accurately

a.) $f(x) = \frac{1}{2}(x+4)^2 - 5$



y-int: (0,3)
vertex: (-4, -5)
Domain: $(-\infty, \infty)$
Range: $[-5, \infty)$

x = -4
axis of
symmetry. x = 2

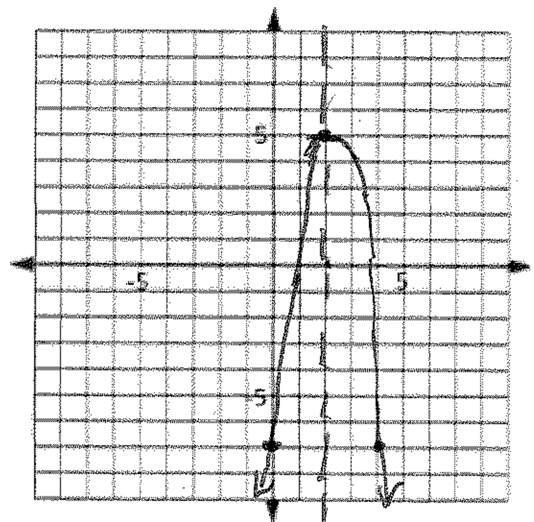
b.) $g(x) = -3(x-2)^2 + 5$

x

↕ by 3

→ by 2   ↑ by 5

y-int: (0, -7)
vertex (2, 5)
Domain $(-\infty, \infty)$
Range: $(-\infty, 5]$



Example 2: Graph $f(x) = x^2 - 8x + 9$ by completing the square

y-int $(0, 9)$

$f(x) = x^2 - 8x + 9$

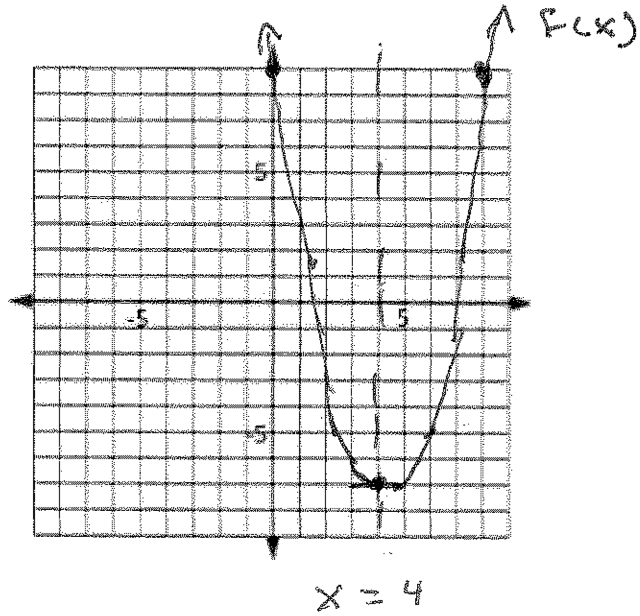
$= (x^2 - 8x \quad) + 9$

$= (x^2 - 8x + 16) + 9 - 16$

$= (x-4)^2 - 7$

vertex $(4, -7)$

Range: $[-7, \infty)$



Example 3: Graph $g(x) = 2x^2 + 4x + 6$ by completing the square

y-int: $(0, 6)$

$g(x) = (2x^2 + 4x) + 6$

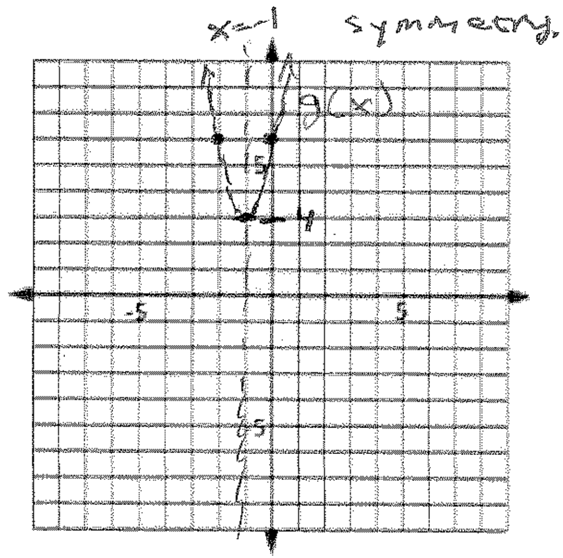
$= 2(x^2 + 2x \quad) + 6$

$= 2(x^2 + 2x + 1) + 6 - 2$

$= 2(x+1)^2 + 4$

vertex $(-1, 4)$

Range $[4, \infty)$



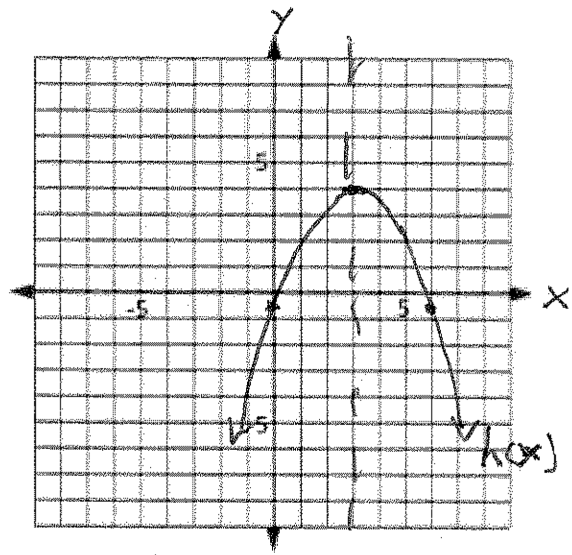
Example 4: Graph $h(x)=-\frac{1}{2}x^2+3x-\frac{1}{2}$ by completing the square

$$h(x)=\left(-\frac{1}{2}x^2+3x\right)-\frac{1}{2}$$
$$=-\frac{1}{2}(x^2-6x)-\frac{1}{2}$$
$$=-\frac{1}{2}(x^2-6x+9)-\frac{1}{2}+\frac{9}{2}$$
$$=-\frac{1}{2}(x-3)^2+4$$

vertex: $(3,4)$

y-int: $(0,-\frac{1}{2})$

Range: $(-\infty,4]$



Find the Formula: Find the vertex of the general quadratic $f(x)=ax^2+bx+c$ by completing the square.

$$f(x)=ax^2+bx+c$$
$$=(ax^2+bx)+c$$
$$=a\left(x^2+\frac{b}{a}x\right)+c \qquad \left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}$$
$$=a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)+c-\frac{b^2}{4a}$$
$$=a\left(x+\frac{b}{2a}\right)^2+c\cdot\frac{4a}{4a}-\frac{b^2}{4a}$$
$$=a\left(x-\left(-\frac{b}{2a}\right)\right)^2+\frac{4ac-b^2}{4a}$$

Vertex: $\left(\frac{-b}{2a},\frac{4ac-b^2}{4a}\right)$

$$\left(-\frac{b}{2a},f\left(-\frac{b}{2a}\right)\right)$$

Method: The vertex of a parabola

a.) The vertex of the parabola given by $f(x) = ax^2 + bx + c$ is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

b.) The longer version of the formula is: $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$

c.) The x-coordinate of the vertex is $-\frac{b}{2a}$. The equation of the axis of symmetry is $x = -\frac{b}{2a}$. The second coordinate of the vertex is most commonly found by computing $f\left(-\frac{b}{2a}\right)$.

Example 5: Find the vertex of $g(x) = 2x^2 + 5x - 1$. Check with your graphing calculator.

$a=2,\ b=5,\ c=-1$

$x = -\frac{b}{2a} = \frac{-5}{4}$

vertex $\left(\frac{-5}{4}, -\frac{33}{8}\right)$

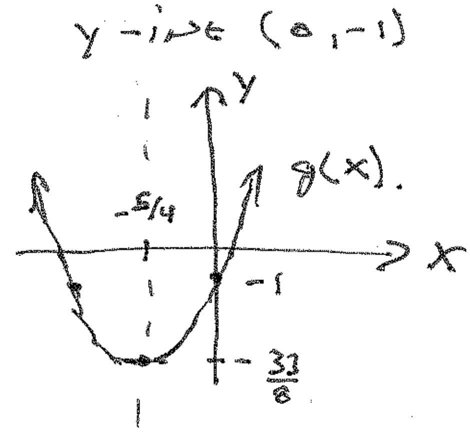
y-int $(0,-1)$

$y = g\left(-\frac{5}{4}\right) = 2\cdot\left(-\frac{5}{4}\right)^2 + 5\left(-\frac{5}{4}\right) - 1$

$= \frac{50}{16} - \frac{25}{4} - 1$

$= \frac{50}{16} - \frac{100}{16} - \frac{16}{16}$

$= -\frac{66}{16} = -\frac{33}{8}$



Example 6: Consider $f(x) = 4x^2 - 12x + 3$. Find the vertex, all intercepts, the min/max value, and the range.

$x = -\frac{-12}{8} = \frac{12}{8} = \frac{3}{2}$

$y = f\left(\frac{3}{2}\right) = -6$

vertex: $\left(\frac{3}{2}, -6\right)$

y-int: $(0,3)$

Range: $[-6, \infty)$

min. @ $y = -6$

x-int.

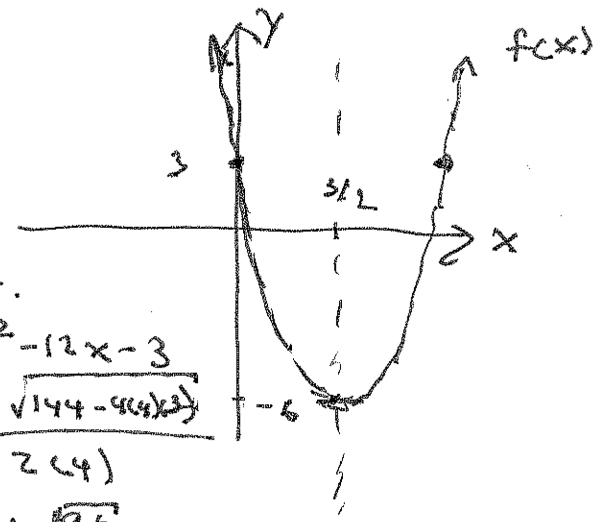
$0 = 4x^2 - 12x - 3$

$\Rightarrow x = \frac{12 \pm \sqrt{144 - 4(4)(3)}}{2(4)}$

$= \frac{12 \pm \sqrt{96}}{8}$

$= \frac{12 \pm 4\sqrt{6}}{8}$

$\Rightarrow x = \frac{3 \pm \sqrt{6}}{2}$



Example 7: Consider $g(x) = -18.8x^2 + 7.92x + 6.18$. Find the vertex, all intercepts, the min/max value, and the range.

vertex

$x = -\dfrac{7.92}{2(-18.8)}$

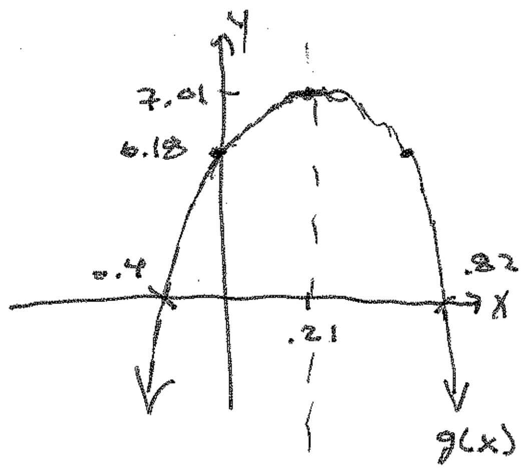
$= 0.21$

$y = g(0.21)$

$= 7.01$

$(0.21, 7.01)$

y-int $(0, 6.18)$



Max: $y = 7.01$

range: $(-\infty, 7.01]$

x-intercepts.

$x = \dfrac{-7.92 \pm \sqrt{7.92^2 - 4(-18.8)(6.18)}}{2(-18.8)}$

$x = -0.40$ or

$x = 0.82$

Summary: The graph of a quadratic equation given by $f(x) = ax^2 + bx + c$ or $f(x) = a(x-h)^2 + k$

a.) The graph is a parabola

b.) The vertex is (h,k) or $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$

c.) The axis of symmetry is $x = h$ or $x = -\dfrac{b}{2a}$

d.) The *y*-intercept of the graph is $(0,c)$

e.) The *x*-intercepts can be found by solving $ax^2 + bx + c = 0$

- a. If $b^2 - 4ac > 0$, there are two real *x*-intercepts
- b. If $b^2 - 4ac = 0$, there is one *x*-intercept
- c. If $b^2 - 4ac < 0$, there are no real *x*-intercepts (although there are two complex zeros)

f.) The domain of the function is $(-\infty, \infty)$

g.) If $a > 0$:

- a. The graph opens upward
- b. The function has a minimum value of *k* at (h,k)
- c. The range of the function is $[k, \infty)$

h.) If $a < 0$:

- a. The graph opens downward
- b. The function has a maximum value of *k* at (h,k)
- c. The range of the function is $(-\infty, k]$