Rational Exponents (7.2) **Math 098**

<u>Definition</u>: $a^{1/n} = \sqrt[n]{a}$. When *a* is nonnegative, *n* can be any natural number greater than 1. When *a* is negative, *n* must be odd.

Example 1: Write in radical notation and simplify.

a.) $x^{1/2}$

$= \sqrt[2]{x}$

$= \sqrt{x}$

b.) $(-27)^{1/3}$

$= \sqrt[3]{-27}$

$= -3$

c.) $(365^{12})^{1/2}$

$= \sqrt{365^{12}}$

$= \sqrt{(365^{6})^{2}}$

$= 365^{6}$

Example 2: Write with exponential notation.

a.) $\sqrt[4]{7ab}$

$= (7ab)^{1/4}$

b.) $\sqrt[5]{\dfrac{3x}{7y}}$

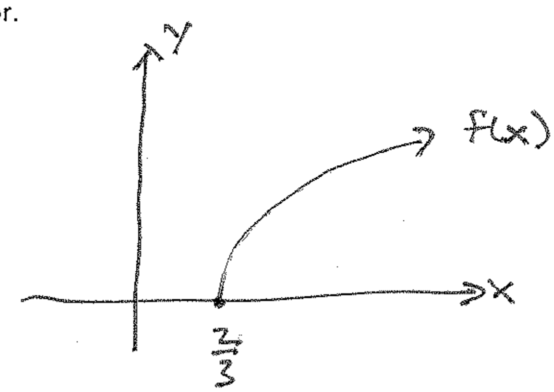
$= \left(\dfrac{3x}{7y}\right)^{1/5}$

Example 3: Graph $f(x) = \sqrt[4]{3x-2}$ on your calculator.

$= (3x-2)^{1/4}$

Domain: $[\frac{2}{3}, \infty)$

Range: $[0, \infty)$



Definition: (Positive rational exponents) For any natural numbers m and n ($n \neq 0$) and any real number a for which $\sqrt[n]{a}$ exists, we have that $a^{m/n}$ means $\left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$

Example 4: Write in radical notation and simplify

a.) $8^{2/3}$

$\sqrt[3]{8^2} = \sqrt[3]{64} = 4$

$\left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$

b.) $36^{3/2}$

$\sqrt{36^3} = \sqrt{46656} = 216$

$\left(\sqrt{36}\right)^3 = (6)^3 = 216$

Definition: (Negative rational exponents) For any rational number m/n and any nonzero real number a for which $a^{m/n}$ exists, we have that $a^{-m/n}$ means $\frac{1}{a^{m/n}}$.

Example 5: Write with positive exponents and simplify if possible.

a.) $49^{-1/2} = \frac{1}{49^{1/2}}$

$= \frac{1}{\sqrt{49}}$

$= \frac{1}{7}$

b.) $(-27)^{-2/3} = \frac{1}{(-27)^{2/3}}$

$= \left(\frac{1}{\sqrt[3]{-27}}\right)^2$

$= \frac{1}{(-3)^2}$

$= \frac{1}{9}$

c.) $5a^{-3/2}b^{4/3} = \frac{5b^{4/3}}{a^{3/2}}$

d.) $\left(\frac{x}{y}\right)^{-3/5}$

$= \frac{1}{\left(\frac{x}{y}\right)^{3/5}}$

$= \left(\frac{y}{x}\right)^{3/5}$

Definition: (Laws of exponents) For any real numbers a and b and any rational exponents m and n for which a^m, a^n, and b^m are defined:

1.) $a^m \cdot a^n = a^{m+n}$ In multiplying, add exponents if the bases are the same.

2.) $\dfrac{a^m}{a^n} = a^{m-n}$ In dividing, subtract exponents if the bases are the same. Assume $a \neq 0$.

3.) $\left(a^m\right)^n = a^{m \cdot n}$ To raise a power to a power, multiply the exponents.

4.) $\left(ab\right)^m = a^m b^m$ To raise a product to a power, raise each factor to the power and multiply.

Example 6: Simplify (answers should have positive exponents)

a.) $5^{3/7} \cdot 5^{1/7} = 5^{\frac{3}{7}+\frac{1}{7}}$
$= 5^{4/7}$

b.) $\dfrac{a^{1/6}}{a^{1/2}} = a^{\frac{1}{6}-\frac{1}{2}}$
$= a^{-\frac{2}{6}}$
$= a^{-\frac{1}{3}} = \dfrac{1}{a^{1/3}}$

c.) $\left(\pi^{3/4}\right)^{2/3} = \pi^{\frac{6}{12}}$
$= \pi^{\frac{1}{2}}$
$= \sqrt{\pi}$

d.) $\left(r^{-1/4}\, b^{3/7}\right)^{1/3} = r^{-\frac{1}{12}}\, b^{\frac{3}{21}}$
$= \dfrac{b^{1/7}}{r^{1/12}}$

Method: To simplify radical expressions

1.) Convert radical expressions to exponential expressions.
2.) Use arithmetic and the laws of exponents to simplify.
3.) Convert back to radical notation as needed.

Example 7: Simplify

a.) $\sqrt[4]{s^{12}} = (s^{12})^{\frac{1}{4}}$
$= s^3$

b.) $\left(\sqrt[5]{x^2 y}\right)^{20} = \left((x^2y)^{1/5}\right)^{20}$
$= (x^2y)^4$
$= x^8 y^4$

c.) $\sqrt[8]{(3y)^4} = \left((3y)^4\right)^{\frac{1}{8}}$
$= (3y)^{1/2}$
$= \sqrt{3y}$

d.) $\sqrt[3]{\sqrt{r}}$
$= \left(r^{1/2}\right)^{1/3}$
$= r^{1/6}$
$= \sqrt[6]{r}$