$3^2 = 9$, so 3 is a square root of 9.

$(-3)^2 = 9$, so -3 is a square root of 9.

Definition: The number c is a square root of a if $c^2 = a$

Example 1: Find the square roots of 49.

$49 = 7^2 \longrightarrow \text{sqrt} = 7$ principle square root.

$49 = (-7)^2 \longrightarrow \text{sqrt} = -7$

Definition: The *principle square root* of a nonnegative number is its nonnegative square root. The symbol $\sqrt{\ }$ is called a *radical sign* and is used to indicate the principal square root of a number over which it appears.

Example 2: Simplify

a.) $\sqrt{36} = +6$

b.) $\sqrt{0.64} = .8$

c.) $-\sqrt{121} = -11$

d.) $\sqrt{40}$ Not a perfect square.

e.) $\sqrt{\frac{25}{81}} = \frac{5}{9}$

Perfect Squares

$1^2 = 1$	$2^2 = 4$
$3^2 = 9$	$4^2 = 16$
$5^2 = 25$	$6^2 = 36$
$7^2 = 49$	$8^2 = 64$
$9^2 = 81$	$10^2 = 100$
$11^2 = 121$	$12^2 = 144$
$13^2 = 169$	$14^2 = 196$
$15^2 = 225$	$16^2 = 256$
$17^2 = 289$	$18^2 = 324$
$19^2 = 361$	$20^2 = 400$

Definition: Any expression containing radicals is a radical expression.

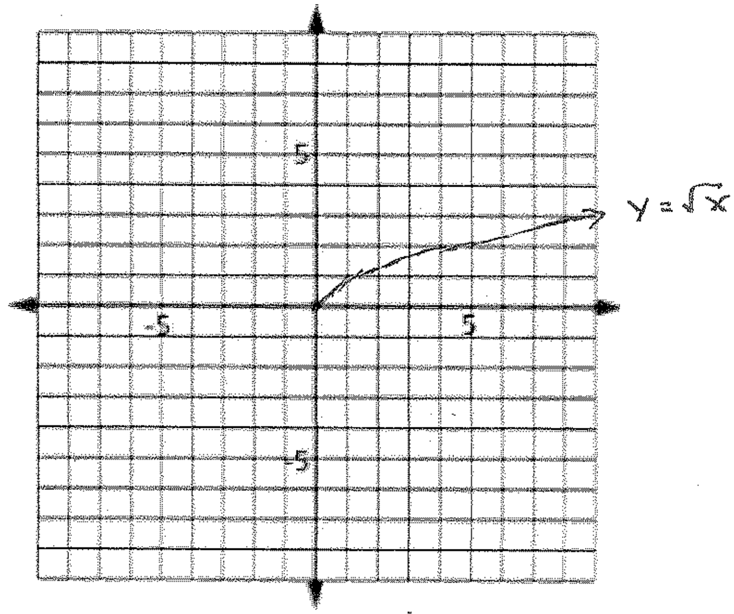
Example 3: Let's graph $f(x)=\sqrt{x}$

Table | Domain | Graph

x	y
Neg x	undefined
0	0
1	1
2	1.41...
3	1.73...
4	2
5	2.23...

Domain
$x \geq 0$
$[0, \infty)$

Range
$y \geq 0$
$[0, \infty)$



Example 4: Consider the functions $f(x)=\sqrt{4-x}$ and $g(x)=-\sqrt{2x-3}$.

a.) Evaluate $f(-5)$

$f(-5)=\sqrt{4-(-5)}=\sqrt{9}=3$

b.) Evaluate $g(2)$

$g(2)=-\sqrt{2(2)-3}=-\sqrt{1}=-1$

c.) What is the domain of *f*?

$4-x \geq 0$

$\Rightarrow -x \geq -4$

$\Rightarrow x \leq 4$

$(-\infty, 4]$

d.) What is the domain of *g*?

$2x-3 \geq 0$

$\Rightarrow 2x \geq 3$

$\Rightarrow x \geq \frac{3}{2}$

$[\frac{3}{2}, \infty)$

Example 5: Evaluate (carefully)

a.) $\sqrt{4^2}$

$=\sqrt{16}$

$=|4|$

$=4$

b.) $\sqrt{(-4)^2}$

$=\sqrt{16}$

$=4$

c.) $\sqrt{a^2}$

$=|a|$

Definition: For any real number a, $\sqrt{a^2} = |a|$. That is, the principal square root of a^2 is the absolute value of a.

Example 6: Simplify

a.) $\sqrt{(x+3)^2}$

$= |x+3|$

b.) $\sqrt{4x^2 - 12x + 9}$

$= \sqrt{(2x + 3)^2}$

$= |2x - 3|$

c.) $\sqrt{r^{12}}$

$= \sqrt{(r^6)^2}$

$= |r^6|$

$= r^6$

odd/even power

d.) $\sqrt{t^{10}}$

$= \sqrt{(t^5)^2}$

$= |t^5|$

Example 7: Simplify, assuming the variables are non-negative

a.) $\sqrt{y^6} = y^3, y \geq 0$

b.) $\sqrt{25x^2 - 10x + 1}$

$= \sqrt{(5x-1)^2}$

$= |5x-1|, x \geq 0$

So far we have been strictly interested in squares and square roots. Now let's broaden our scope.

$3^3 = \cancel{9}\ 27$, so 3 is a cube root of 27.

$(-3)^3 = -27$, so -3 is a cube root of 27.

Definition: The number c is the cube root of a if $c^3 = a$. In symbols, we write $\sqrt[3]{a}$ to denote the cube root of a.

Example 8: Let's graph $f(x)=\sqrt[3]{x}$

Table

x	y
−27	−3
−8	−2
−1	−1
0	0
1	1
8	2
27	3

Domain

$x \in \mathbb{R}$

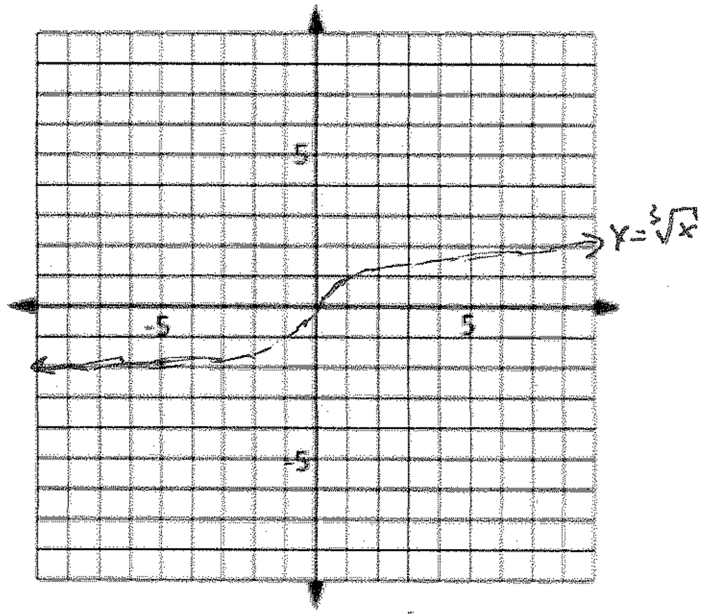
$(-\infty, \infty)$

Range

$y \in \mathbb{R}$

$(-\infty, \infty)$

Graph



If $b^n = a$, then b is the ___$n$th___ ___root___ of a.

Example 9: Evaluate

a.) $\sqrt[3]{216} = 6$

b.) $\sqrt[4]{81} = \cancel{81}\ 3$

Perfect Cubes

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

$6^3 = 216$

c.) $\sqrt[3]{-64} = -4$

d.) $-\sqrt[5]{-32} = -(-2)$

$= 2$

e.) $\sqrt[4]{-81}$ Not real.

f.) $\sqrt[5]{r^5} = r$

odd pwr

so no ABS

g.) $\sqrt[6]{x^6} = |x|$

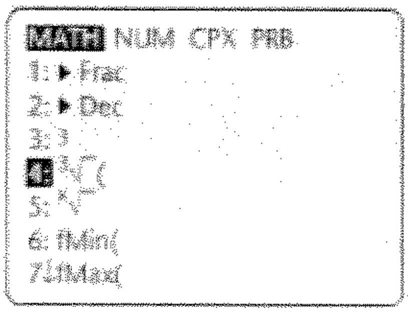
even pwr

so ABS

Method: Simplifying n^{th} roots

n	a	$\sqrt[n]{a}$	$\sqrt[n]{a^n}$
Even	Positive	Positive	$\lvert a\rvert$ (or a)
	Negative	Not a real number	$\lvert a\rvert$ (or $-a$)
Odd	Positive	Positive	a
	Negative	Negative	a

Now that we understand radicals, let's focus on radical functions – functions that can be described by radical expressions.



Be very careful when entering roots into your calculator.

$\sqrt[3]{64} = 4$

$\sqrt[5]{-32} = -2$

Example 10: Find the domain of the given functions algebraically, then use the graph to determine the range.

a.) $f(x)=\sqrt{-x}$

non-negative

Domain

$-x \geq 0$

$x \leq 0$

$(-\infty, 0]$

$\{x \mid x \leq 0\}$

Range

$y \geq 0$

$[0, \infty)$

$\{y \mid y \geq 0\}$.

b.) $g(x)=\sqrt{4x-3}-2$

Domain

$4x-3 \geq 0$

$\Rightarrow 4x \geq 3$

$\Rightarrow x \geq 3/4$

$[3/4, \infty)$

$\{x \mid x \geq 3/4\}$.

Range

$y \geq -2$

$[-2, \infty)$

$\{y \mid y \geq -2\}$.

c.) $r(x)=\sqrt{x^2+1}$

Domain

$x^2+1 \geq 0$

$\Rightarrow x^2 \geq -1$

$\Rightarrow x \in \mathbb{R}$

$(-\infty, \infty)$

$\{x \mid x \in \mathbb{R}\}$

Range.

$y \geq 1$

$[1, \infty)$

$\{y \mid y \geq 1\}$.

d.) $s(x)=\sqrt[4]{5-2x}$

Domain

$5-2x \geq 0$

$\Rightarrow 5 \geq 2x$

$\Rightarrow \frac{5}{2} \geq x$

$(-\infty, \frac{5}{2}]$

$\{x \mid x \leq \frac{5}{2}\}$.

Range

$y \geq 0$

$[0, \infty)$

$\{y \mid y \geq 0\}$.

Example 11: Determine whether a radical function would be a good model (eye ball the model).

a.) The following table lists the average size of United States' farms for various years from 1940 to 2002

Year	Average Farm Size (in acres)
1940	175
1960	303
1980	426
1997	431
2002	441

b.) The following table lists the amount of federal funds allotted to the National Cancer Institute for cancer research in the United States from 2003 to 2007.

Year	Funds (in billions)
2003	4.59
2004	4.74
2005	4.83
2006	4.79
2007	4.75