Method: To solve a rational equation

Multiply both sides of the equation by the LCD. This is called *clearing fractions* and produces an equation similar to those we have already solved. After solving this equation, make sure to check your result for extraneous solutions.

Example 1: Solve LCD = $2t$

a.) $\frac{1}{2}-\frac{2}{t}=\frac{3}{2t}$

$\Rightarrow 2t\left(\frac{1}{2}-\frac{2}{t}\right)=2t\left(\frac{3}{2t}\right)$

$\Rightarrow t-4=3$

$\Rightarrow t=7$ ✓ check.

b.) $\frac{x-2}{x-4}=\frac{2}{x-4}$ (after solving, observe the graphs) LCD = $x-4$

$\Rightarrow (x-4)\cdot\frac{x-2}{x-4}=(x-4)\cdot\frac{2}{x-4}$

"No solution"

$\Rightarrow x-2=2$

$\Rightarrow x=4$ ← extraneous solution.

c.) $\frac{x}{x+1}+\frac{5}{x}=\frac{1}{x^2+x}$ LCD = $x(x+1)$

$\Rightarrow \frac{x}{x+1}+\frac{5}{x}=\frac{1}{x(x+1)}$

$\Rightarrow x(x+1)\left(\frac{x}{x+1}+\frac{5}{x}\right)=\frac{1}{x(x+1)}\cdot x(x+1)$

$\Rightarrow x^2+5(x+1)=1$

$\Rightarrow x^2+5x+5=1$

$\Rightarrow x^2+5x+4=0$

$\Rightarrow (x+4)(x+1)=0$

$\Rightarrow \boxed{x=-4}$ OR $x=-1$

d.) $\frac{3-2y}{y+1} - \frac{10}{y^2-1} = \frac{2y+3}{1-y}$ $\quad LCD = (y+1)(y-1)$

$\Rightarrow \quad \frac{3-2y}{y+1} - \frac{10}{(y+1)(y-1)} = -\frac{2y+3}{y-1}$

$\Rightarrow \quad (3-2y)(y-1) - 10 = -(2y+3)(y+1)$

$\Rightarrow \quad 3y - 3 - 2y^2 + 2y - 10 = -2y^2 - 2y - 3y - 3$

$\Rightarrow \quad 10y = 10$

$\Rightarrow \quad y = 1$

no solution.

Example 2: Consider $f(x) = \frac{3x-1}{x^2-7x+10}$ and $g(x) = \frac{x-1}{x^2-4} + \frac{2x+1}{x^2-3x-10}$. Find all values of *a* such that $f(a) = g(a)$.

$f(a) = g(a)$

$\frac{3a-1}{a^2-7a+10} = \frac{a-1}{a^2-4} + \frac{2a+1}{a^2-3a-10}$ $\quad LCD = (a-5)(a-2)(a+2)$

$\Rightarrow \quad \frac{3a-1}{(a-5)(a-2)} = \frac{a-1}{(a-2)(a+2)} + \frac{2a+1}{(a-5)(a+2)}$

$\Rightarrow \quad (3a-1)(a+2) = (a-1)(a-5) + (2a+1)(a-2)$

$\Rightarrow \quad 3a^2 + 5a - 2 = a^2 - 6a + 5 + 2a^2 - 3a - 2$

$\Rightarrow \quad 14a = 5$

$\Rightarrow \quad a = \frac{5}{14}$

Example 3: Solve

a.) $\frac{3}{x}+\frac{x}{x+2}=\frac{4}{x^2+2x}$

LCD $= x(x+2)$

$\Rightarrow \frac{3}{x} + \frac{x}{x+2} = \frac{4}{x(x+2)}$

multiply both sides by LCD

$\Rightarrow 3(x+2) + x \cdot x = 4$

$x(x+2)\left(\frac{3}{x} + \frac{x}{x+2}\right) = x\cdot(x+2)\frac{4}{x(x+2)}$

$\Rightarrow 3x + 6 + x^2 = 4$

$\Rightarrow 3(x+2) + x^2 = 4$

$\Rightarrow x^2 + 3x + 2 = 0$

$\Rightarrow (x+2)(x+1) = 0$

$\Rightarrow$ ~~$x = -2$~~ OR $\boxed{x = -1}$

b.) $\frac{y+3}{y+2}-\frac{y}{y^2-4}=\frac{y}{y-2}$

LCD $= (y+2)(y-2)$

$\Rightarrow \frac{y+3}{y+2} - \frac{y}{(y+2)(y-2)} = \frac{y}{y-2}$

$\Rightarrow (y+2)(y-2)\left\{\frac{y+3}{y+2} - \frac{y}{(y+2)(y-2)}\right\} = (y+2)(y-2)\cdot\frac{y}{y-2}$

$\Rightarrow (y-2)(y+3) - y = y(y+2)$

$\Rightarrow y^2 + y - 6 - y = y^2 + 2y$

$-y^2 \qquad\qquad -y^2$

$\Rightarrow -6 = 2y$

$\Rightarrow y = -3$

Example 4: Find all values of *x* for which the rational function $g(x)=\frac{x-3}{x+2}$ is equal to $\frac{1}{5}$

$$\frac{x-3}{x+2} = \frac{1}{5}$$

cross multiply

$$\Rightarrow 5(x-3) = 1\cdot(x+2)$$

$$\Rightarrow 5x-15 = x+2$$

$$\Rightarrow 4x = 17$$

$$\Rightarrow x = \frac{17}{4}$$

Example 5: For the functions $f(x)=\frac{x+4}{3x}$ and $g(x)=2-\frac{x+8}{5x}$, find all values of *a* for which $f(a)=g(a)$.

$f(a) = g(a)$ LCD = $15a$

$$\frac{a+4}{3a} = 2-\frac{a+8}{5a}$$

$$\Rightarrow 5(a+4) = 2(15a) - 3(a+8)$$

$$\Rightarrow 5a+20 = 30a - 3a - 24$$
$$-5a \quad -5a$$
$$+24 \quad +24$$

$$\Rightarrow 44 = 22a$$

$$\Rightarrow a = 2$$