Definition: A complex rational expression contains ___rational___ ___expressions___

within its numerator and/or denominator.

We will use two different methods for simplifying complex rational expressions. **DO NOT** mix the methods.

$$LCD = rt$$

Example 1: Simplify

a.) $\dfrac{\dfrac{2}{r}-\dfrac{3}{t}}{\dfrac{4}{r}-\dfrac{5}{t}} \cdot \dfrac{rt}{rt} = \dfrac{2t-3r}{4t-5r}$

$$LCD = a^3 b^2$$

b.) $\dfrac{\dfrac{1}{a^3 b}+\dfrac{1}{b}}{\dfrac{1}{a^2 b^2}-\dfrac{1}{b^2}} \cdot \dfrac{a^3 b^2}{a^3 b^2} = \dfrac{b+a^3 b}{a-a^3}$

$$= \dfrac{b(1+a^3)}{a(1-a^2)}$$

$$= \dfrac{b(1+a)(1-a+a^2)}{a(1-a)(1+a)}$$

$$= \dfrac{b(1-a+a^2)}{a(1-a)}$$

Method: Using multiplication by 1 to simplify a complex rational expression.

1. Find the LCD of all rational expressions within the complex rational expression.
2. Multiply the complex rational expression by 1, writing 1 as the LCD divided by itself.
3. Distribute and simplify so that the numerator and the denominator of the complex rational expression are polynomials.
4. Factor and, if possible, simplify.

Example 2: Simplify

a.) $\dfrac{\dfrac{3}{a^2-9}+\dfrac{2}{a+3}}{\dfrac{4}{a^2-9}+\dfrac{1}{a+3}}$ $=$ $\dfrac{\dfrac{3}{(a+3)(a-3)}+\dfrac{2}{a+3}}{\dfrac{4}{(a+3)(a-3)}+\dfrac{1}{a+3}}$ $\cdot$ $\dfrac{\dfrac{(a+3)(a-3)}{1}}{\dfrac{(a+3)(a-3)}{1}}$

$$= \dfrac{3+2(a-3)}{4+1(a-3)}$$

$$= \dfrac{2a-3}{a+1}$$

b.) $\dfrac{\dfrac{3x}{y}-x}{2y-\dfrac{y}{x}}$ $\cdot$ $\dfrac{xy}{xy}$ $\qquad$ LCD $= xy$ $\qquad = \dfrac{3x^2-x^2y}{2xy^2-y^2}$

$$= \dfrac{x^2(3-y)}{y^2(2x-1)}$$

Method: Using division to simplify a complex rational expression.

1.) Add or subtract, as necessary, to get one rational expression in the numerator.
2.) Add or subtract, as necessary, to get one rational expression in the denominator.
3.) Perform the indicated division (invert the divisor and multiply).
4.) Simplify, if possible, by removing any factors that equal 1.

Example 3: Simplify

a.) $\dfrac{\dfrac{3x}{y} - x}{2y - \dfrac{y}{x}}$ $\rightleftarrows$ $\dfrac{\dfrac{3x - xy}{y}}{\dfrac{2xy - y}{x}}$

$= \dfrac{3x - xy}{y} \cdot \dfrac{x}{2xy - y}$

$= \dfrac{3x^2 - x^2 y}{2xy^2 - y^2}$

$= \dfrac{x^2(3 - y)}{y^2(2x - 1)}$

b.) $\dfrac{\dfrac{x^2 - x - 12}{x^2 - 2x - 15}}{\dfrac{x^2 + 8x + 12}{x^2 - 5x - 14}}$

$= \dfrac{x^2 - x - 12}{x^2 - 2x - 15} \cdot \dfrac{x^2 - 5x - 14}{x^2 + 8x + 12}$

$= \dfrac{(x+3)(x-4)(x-7)(x+2)}{(x-5)(x+3)(x+6)(x+2)}$

$= \dfrac{(x-4)(x-7)}{(x-5)(x+6)}$, $x \neq -3, -2$

c.) $\dfrac{a^{-1}+b^{-1}}{\dfrac{a^2-b^2}{ab}}$ $= \dfrac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{a^2-b^2}{ab}} \cdot \dfrac{ab}{ab}$

$= \dfrac{b+a}{a^2-b^2}$

$= \dfrac{b+a}{(a+b)(a-b)}$

$= \dfrac{1}{a-b}$

d.) $\dfrac{\dfrac{y}{y^2-1}-\dfrac{3y}{y^2+5y+4}}{\dfrac{3y}{y^2-1}-\dfrac{y}{y^2-4y+3}}$

$LCD = (y+1)(y-1)(y+4)(y-3)$

$= \dfrac{\dfrac{y}{(y+1)(y-1)}-\dfrac{3y}{(y+1)(y+4)}}{\dfrac{3y}{(y+1)(y-1)}-\dfrac{y}{(y-1)(y-3)}} \cdot \dfrac{(y+1)(y-1)(y+4)(y-3)}{(y+1)(y-1)(y+4)(y-3)}$

$= \dfrac{y(y+4)(y-3)-3y(y-1)(y-3)}{3y(y+4)(y-3)-y(y+1)(y+4)}$

$= \dfrac{y(y-3)\{(y+4)-3(y-1)\}}{y(y+4)\{3(y-3)-(y+1)\}}$

$= \dfrac{(y-3)(7-2y)}{(y+4)(2y-10)}$