leading coefficient = 1

Factoring Trinomials: x^2+bx+c **(5.4)** **Math 098**

Let's observe the patterns:

$$(x+3)(x+4) = x^2+4x+3x+12$$

$$= x^2+7x+12 \qquad 3\cdot 4=12 \qquad 3+4=7$$

So to factor x^2+bx+c we look for two numbers that multiply to c and add to b.

Example 1: Factor

a.) $x^2+9x+20$

$4\cdot 5=20$

$4+5=9$

$=(x+4)(x+5)$

same signs

b.) $t^2-12t+32$

$-4(-8)=32$

$-4+(-8)=-12$

$=(t-4)(t-8)$

same signs

So if c is positive, then the two numbers have the same sign, and b determines it.

Example 2: Factor

a.) $r^2 + 5r - 36$

$9(-4) = -36$

$= (r+9)(r-4)$

$9 + (-4) = 5$

b.) $q^2 - 3q - 40$

$5(-8) = -40$

$5 + (-8) = -3$

$= (q+5)(q-8)$

So if *c* is negative, then the two numbers have <u>opposite sign</u>, and *b* will determine which sign will be "<u>bigger</u>" (that is, have more absolute value or weight).

* Important: When factoring always factor out the <u>GCF</u> first.

Example 3: Factor completely

a.) $x^3 + 3x^2 - 4x = x(x^2 + 3x - 4)$

$= x(x+4)(x-1)$

b.) $y^2 + 6y + 15$

Does not factor

"prime"

Example 4: Factor completely

a.) $a^2-2ab-48b^2$

$= (a+6b)(a-8b)$

b.) $2t^2+32t-72$

$= 2(t^2+16t-36)$

$= 2(t+18)(t-2)$

$a^2-2a-48$

$= (a+6)(a-8)$

$\boxed{\text{expression } =}$

Example 5: Solve

a.) $x^2-5x-6=0$

$\Rightarrow (x-6)(x+1)=0$

$\Rightarrow x-6=0$ OR $x+1=0$

$\Rightarrow x=6$ OR $x=-1$.

b.) $(z+4)(z-2)=-5$

$\Rightarrow z^2-2z+4z-8=-5$

$\Rightarrow z^2+2z-3=0$

$\Rightarrow (z+3)(z-1)=0$

$\Rightarrow z=-3$ OR $z=1$

$\boxed{\text{equation } \Rightarrow}$

c.) $2x^5=26x^3-72x$ (graph (c.) when done to observe the roots/zeros. Use the window $[-5,5]\times[-75,75]$)

$\Rightarrow 2x^5-26x^3+72x=0$

$\Rightarrow 2x(x^4-13x^2+36)=0$

$\Rightarrow 2x(x^2-4)(x^2-9)=0$

$\Rightarrow 2x(x+2)(x-2)(x+3)(x-3)=0$

$\Rightarrow x=0, \pm 2, \pm 3$

Example 6: Write a polynomial function $f(x)$ in standard form whose zeros are -3, 0, and 4.

$f(x) = x(x+3)(x-4)$

$= x(x^2 - x - 12)$

$= x^3 - x^2 - 12x$

$g(x) = \frac{12}{7}x(2x+3)(x-4)$

$h(x) = x^5(x+3)^4(x-4)^3$

Example 7: Practice for you (factor or solve as appropriate)

a.) x^2+5x+6

1 · 6

2 · 3

$= (x+2)(x+3)$

b.) x^2-5x-6

-1 · 6

-2 · 3

-3 · 2

-6 · 1

$= (x-6)(x+1)$

c.) x^2-5x+6

-1 · -6

-2 · -3

$= (x-2)(x-3)$

d.) x^2+5x-6

-1 · 6

-2 · 3

-3 · 2

-6 · 1

$= (x-1)(x+6)$

e.) $3r^3 = 45r^2+48r$

$\Rightarrow 3r^3 - 45r^2 - 48r = 0$

$\Rightarrow 3r(r^2 - 15r - 16) = 0$

$\Rightarrow 3r(r-16)(r+1) = 0$

$\Rightarrow r = 0$ or $r = 16$ or $r = -1$

f.) $r^3-3r^2 = 4r-12$

$\Rightarrow r^3 - 3r^2 - 4r + 12 = 0$

$\Rightarrow (r^3 - 3r^2) - (4r - 12) = 0$

$\Rightarrow r^2(r-3) - 4(r-3) = 0$

$\Rightarrow (r-3)(r^2-4) = 0$

$\Rightarrow (r-3)(r-2)(r+2) = 0$

$\Rightarrow r = 3$ or $r = \pm 2$