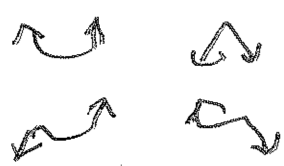
Graph the following functions on your graphing calculator and observe differences between polynomial and non-polynomial functions.

Polynomial Functions	**Non-polynomial Functions**
$f(x)=x^2+3x+5$	$g(x)=\lvert x-4\rvert$
$h(x)=4$	$i(x)=1+\sqrt{2x-5}$
$j(x)=-0.5x^4+5x-2.3$	$k(x)=\dfrac{x-7}{2x}$

smooth

continuous

end behavior



domain

$(-\infty,\infty)$

Polynomial Definitions and Vocabulary

- A number or variable raised to a power or a product of numbers and variables raised to powers is a term. 2^3, x, y^2, x^3y, 2^3xy^2

  x^{-1}, $y^{1/2}$
- A polynomial is one or more terms combined with addition and subtraction. The powers must be non-negative integers

  $0, 1, 2, 3, 4, \ldots$

$3x^2y^4 + 2^4x^2y^4z$

- The degree of a term is the sum of the powers
- The coefficient of a term is the constant (or number) of the term.
- The leading term of a polynomial is the term of highest degree. Its coefficient is the leading coefficient.
- The degree of a polynomial is the degree of the leading term in the polynomial.

Example:

- Types of polynomials (by number of terms):
  - A Monomial is a polynomial with one term.
  - A binomial is a polynomial with two terms.
  - A trinomial is a polynomial with three terms.
- Types of polynomials (by degree):
  - linear if it has degree 0 or 1 $3, 4x, 3+4x$
  - quadratic if it has degree 2 $4-2x^2, x^2+2x-3$
  - cubic if it has degree 3 $1-x^3, 2x^3+4x^2-7$
- The order of a polynomial:
  - ascending order is when the exponents of one variable increase from left to right in the polynomial.
  - descending order is when the exponents of one variable decrease from left to right in the polynomial.

Example 1: For each polynomial, find the degree of each term, the degree of the polynomial, the leading term, and the leading coefficient.

a.) $3x^4 - 17x^2 + 2x - 5$

Term: $3x^4$, $-17x^2$, $2x$, -5

Degree: 4 2 1 0

Leading term: $3x^4$

Leading Coefficient: 3

Degree of the polynomial: 4

b.) $3x^3 - 5x^2y^3 - 8x^4y^2 + 4y^4 + 4x - 7$

Term: $3x^3$, $-5x^2y^3$, $-8x^4y^2$, $4y^4$, $4x$, -7

Degree: 3 5 6 4 1 0

Leading term: $-8x^4y^2$

Leading Coefficient: -8

Degree of the polynomial: 6

Example 2: Arrange the polynomial $3x - 10x^4 + 8 - 3x^2 - 4x^3$ in both ascending and descending order.

Ascending: $8 + 3x - 3x^2 - 4x^3 - 10x^4$

* Descending: $-10x^4 - 4x^3 - 3x^2 + 3x + 8$

A polynomial function has the form $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ where each a_i is a constant and *n* is a non-negative integer.

Example 3: Find $P(-3)$ for $P(x) = -x^2 - 5x + 2$ by hand, evaluating with the calculator, using the table, and by looking at the graph.

$$P(-3) = -(-3)^2 - 5(-3) + 2$$

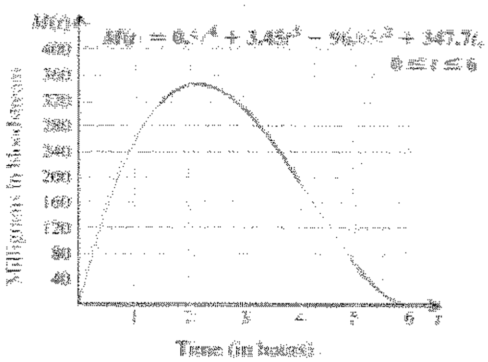
$$= -9 + 15 + 2$$

$$= 8$$

Example 4: Ibuprofen is a medication used to relieve pain. We can estimate the number of milligrams of ibuprofen in the bloodstream *t* hours after 400 mg of medication has been swallowed with the polynomial function $M(t) = 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t,\ 0 \le t \le 6$.

a.) How many milligrams of ibuprofen are in the bloodstream 2 hours after 400 mg has been swallowed?

b.) Use the graph to find and interpret $M(4)$



Fact about polynomials: The domain of the previous example was limited to six hours because of the application. However, the *domain* of every polynomial is $(-\infty, \infty)$ (provided there aren't restrictions added on).

Example 5: Find the domain and range of the following polynomials

a.) $f(x) = x^3 - 3x^2 + 6$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

odd degree

b.) $g(x) = x^4 - 4x^2 + 5$

Domain: $(-\infty, \infty)$

Range: $[1, \infty)$

Example 6: Combine like terms

a.) $3t^2 - 4t - 4t^2 - 3t + 8$

$= -t^2 - 7t + 8$

b.) $5x^2y - 6xy^2 + 2x^2y^2 + 9xy^2 - 9x^2y$

$= -4x^2y + 3xy^2 + 2x^2y^2$

Example 7: Add or subtract polynomials

a.) $(2x^3 - 4x^2 + 5) + (3x^3 - 5x - 3)$

$= 2x^3 - 4x^2 + 5 + 3x^3 - 5x - 3$

$= 5x^3 - 4x^2 - 5x + 2$

b.) $(4s^3 - 7s^2 + 3s + 8) + (-3s^3 - 2s^2 - 5s + 2)$

$= s^3 - 9s^2 - 2s + 10$

c.) $(4x^2y - 7xy + 3y) + (x^2y - 2xy - 7y)$

$= 4x^2y - 7xy + 3y + x^2y - 2xy - 7y$

$= 5x^2y - 9xy - 4y$

d.) $(3t^2 - 4t - 8) - (t^2 + 2t - 5)$

$= 3t^2 - 4t - 8 - t^2 - 2t + 5$

$= 2t^2 - 6t - 3$

e.) $(-4r^3 + 3r - 7) - (3r^2 - 5r + 4)$

$= -4r^3 - 3r^2 + 8r - 11$

f.) $(4x^2y - 7xy + 3y) - (x^2y - 2xy - 7y)$

$= 4x^2y - 7xy + 3y - x^2y + 2xy + 7y$

$= 3x^2y - 5xy + 10y$