4.5: Curve Sketching

ex1: sketch $f(x) = x^4 - 4x^3 + 10$

y-int $(0, 10)$

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$$

min $(3, -17)$

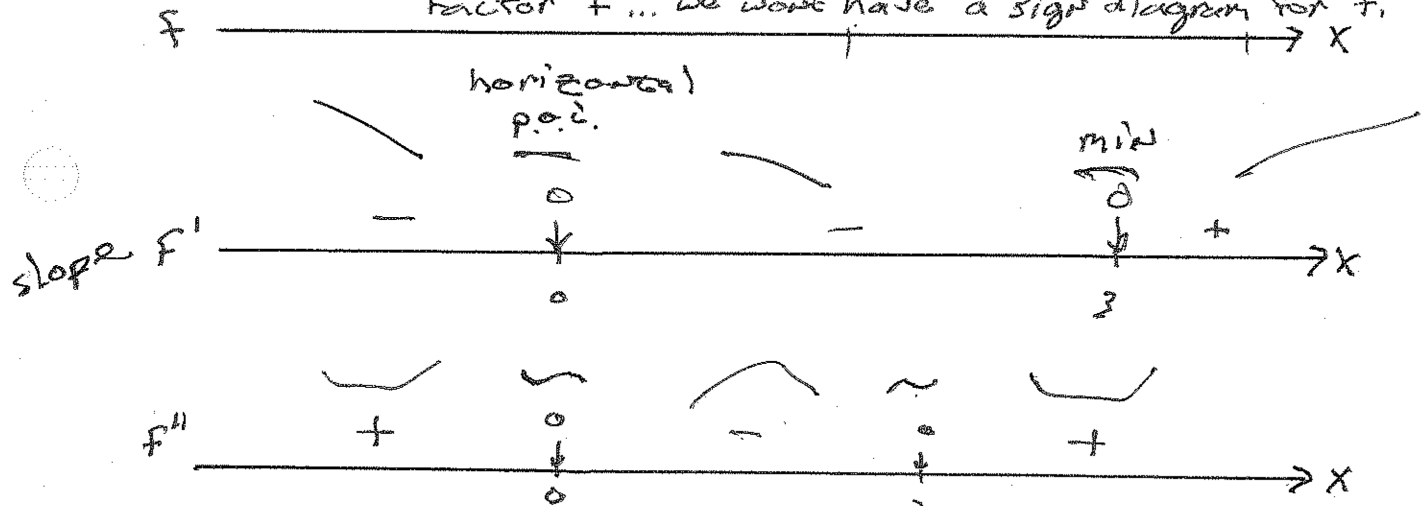
hpoi $(0, 10)$

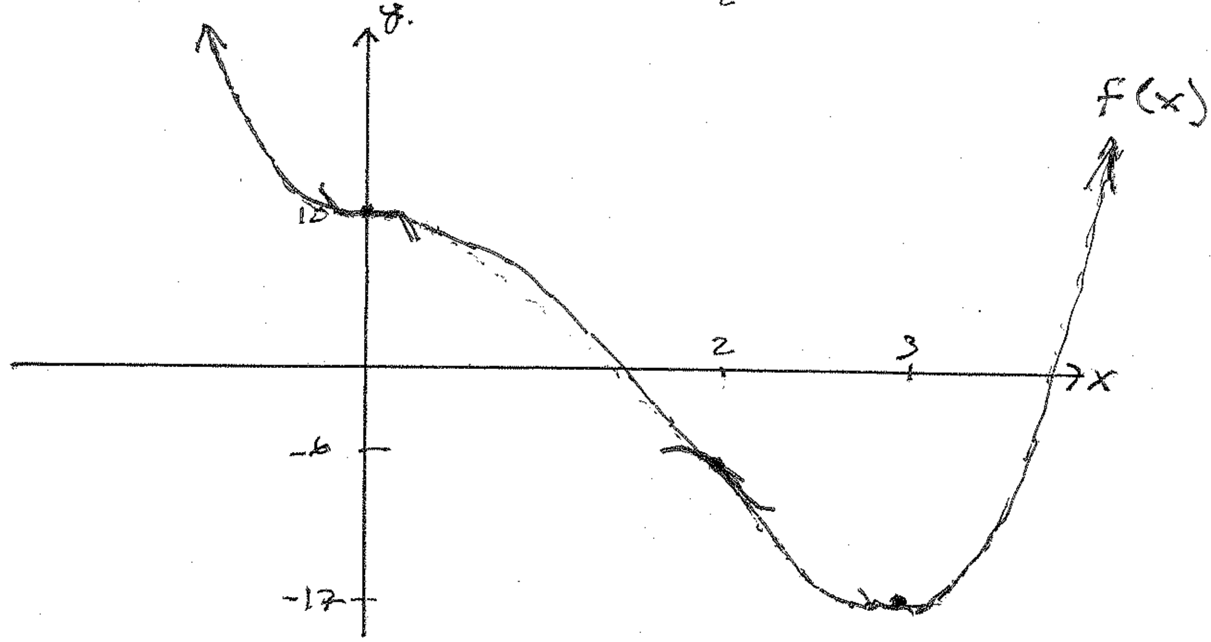
$$f''(x) = 12x^2 - 24x = 12x(x-2)$$

poi $(2, -6)$

end behavior is like $y = x^4$

Note: since we can't easily factor f ... we won't have a sign diagram for f.





ex2: sketch $g(x) = x^3 - 27x$

$$= x(x^2 - 27)$$

$$= x(x + \sqrt{27})(x - \sqrt{27})$$

$$g'(x) = 3x^2 - 27$$

$$= 3(x^2 - 9)$$

$$= 3(x+3)(x-3)$$

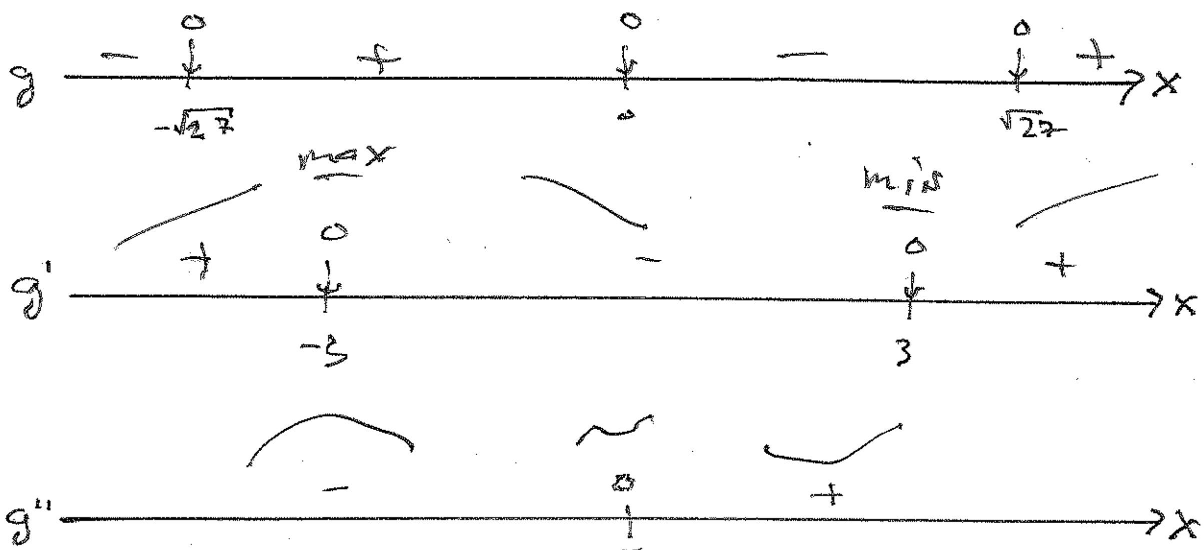
$$g''(x) = 6x$$

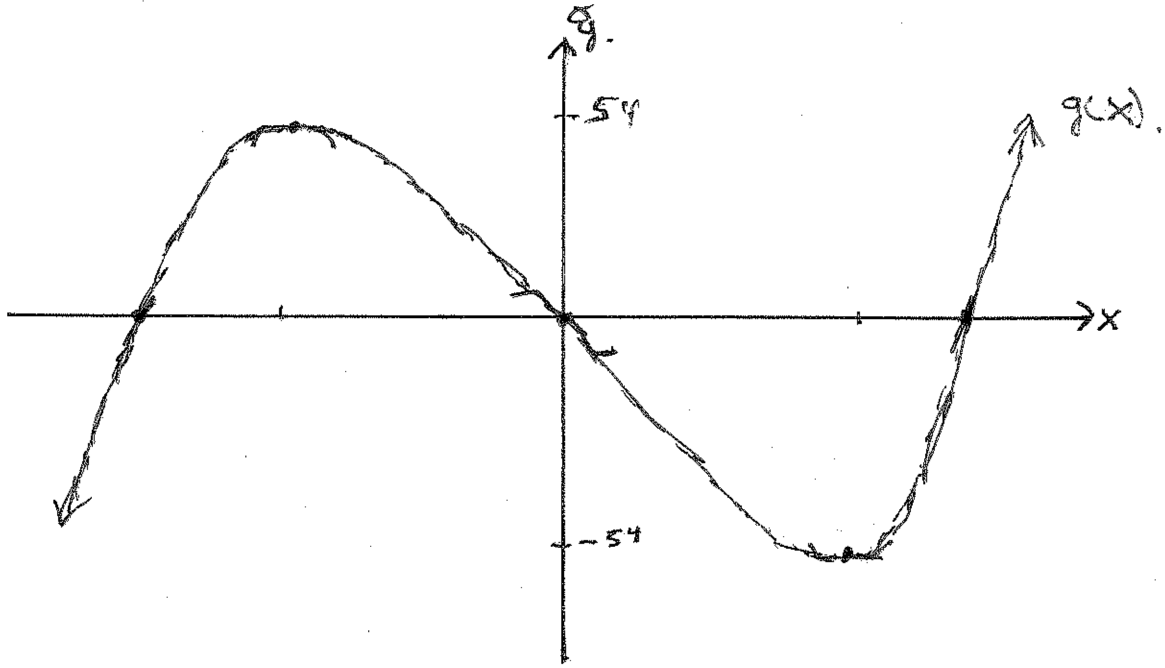
max $(-3, 54)$

min $(3, -54)$

poi $(0, 0)$

endbehavior like $y = x^3$





ex 3: sketch $h(x) = \dfrac{(x+1)^2}{1+x^2}$

$$h'(x) = \frac{-2(x+1)(x-1)}{(1+x^2)^2}$$

$$h''(x) = \frac{4x(x+\sqrt{3})(x-\sqrt{3})}{(1+x^2)^3}$$

min $(-1, 0)$

max $(1, 2)$

poi $(+\sqrt{3}, 1.866)$

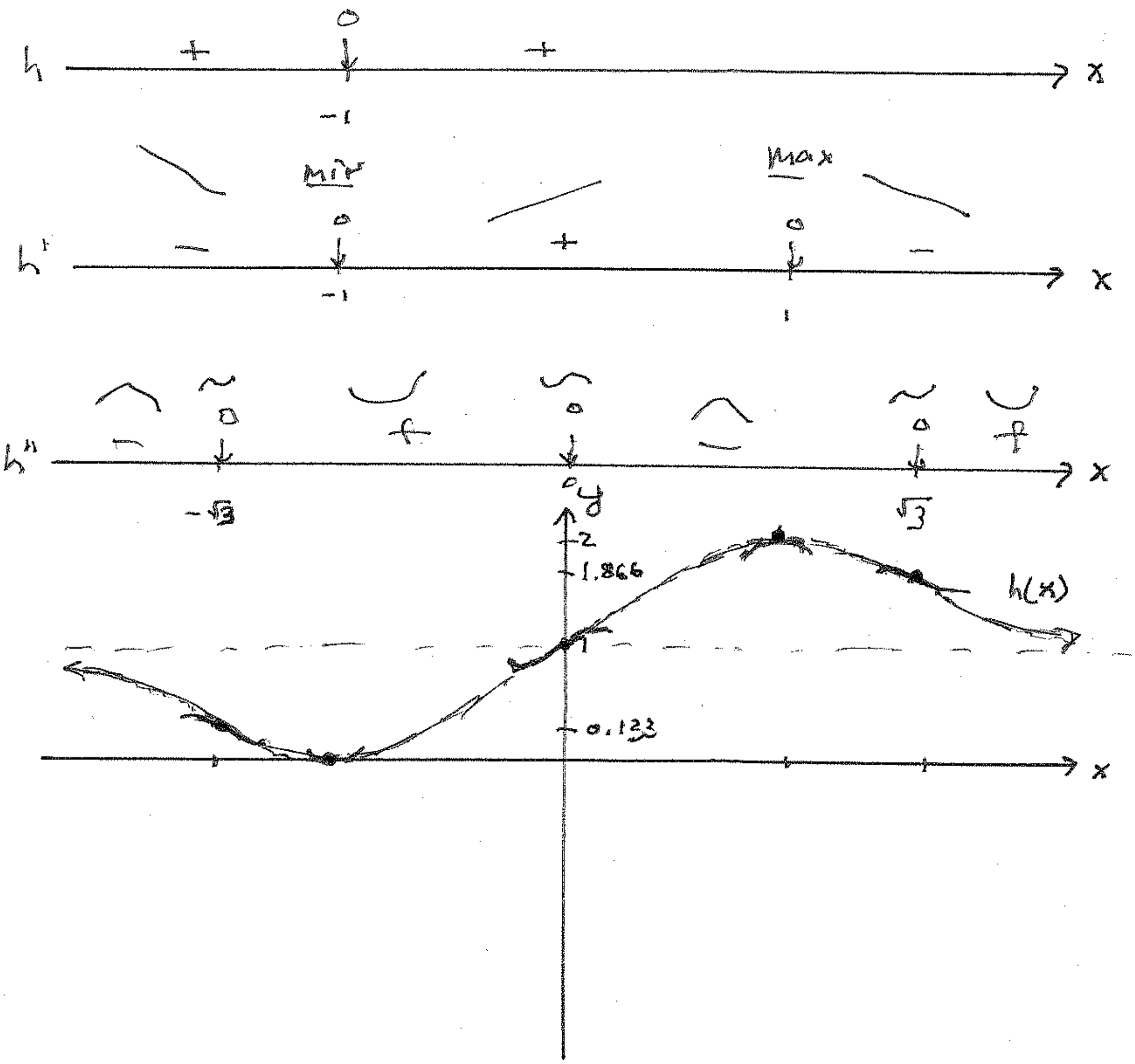
poi $(0, 1)$

poi $(-\sqrt{3}, 0.133)$

endbehavior.

$$\lim_{x\to\infty} h(x) = \lim_{x\to\infty} \frac{(x+1)^2}{1+x^2} = 1$$

horizontal asymptote @ $y=1$.



ex4: sketch $f(x) = x\sqrt{2-x^2}$

$2 - x^2 = (\sqrt{2}+x)(\sqrt{2}-x)$

$f(x) = x\sqrt{(\sqrt{2}+x)(\sqrt{2}-x)}$

check symmetry.

$f(-x) = -x\sqrt{2-(-x)^2}$

$= -x\sqrt{2-x^2}$

$= -f(x)$. f is an odd fcn. Symmetric about origin.

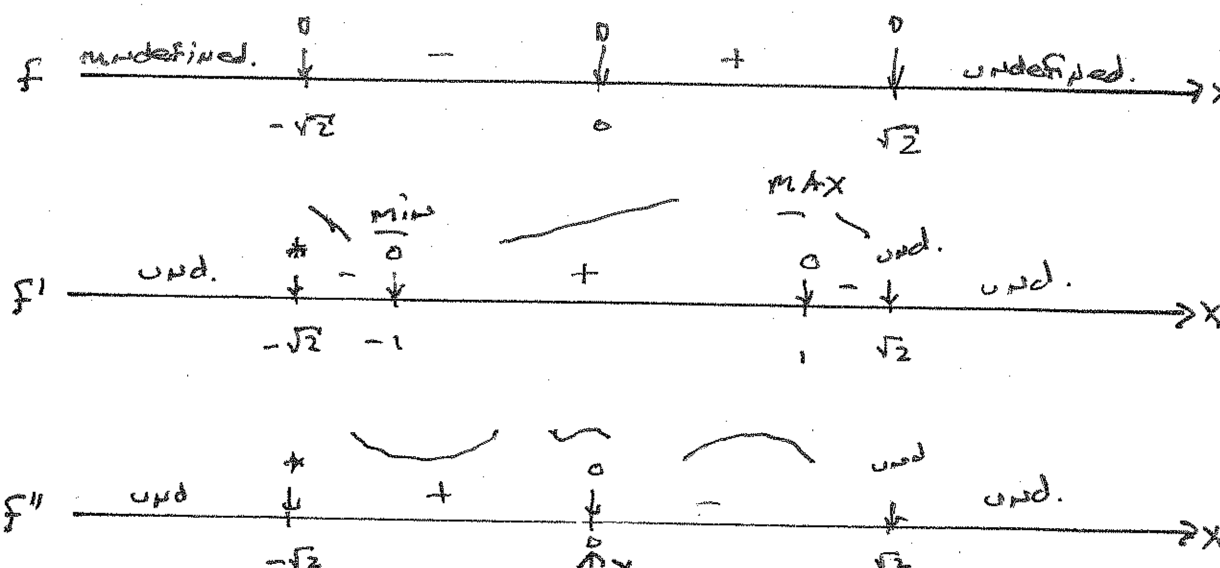
$f'(x) = -\dfrac{2(x^2-1)}{\sqrt{2-x^2}}$

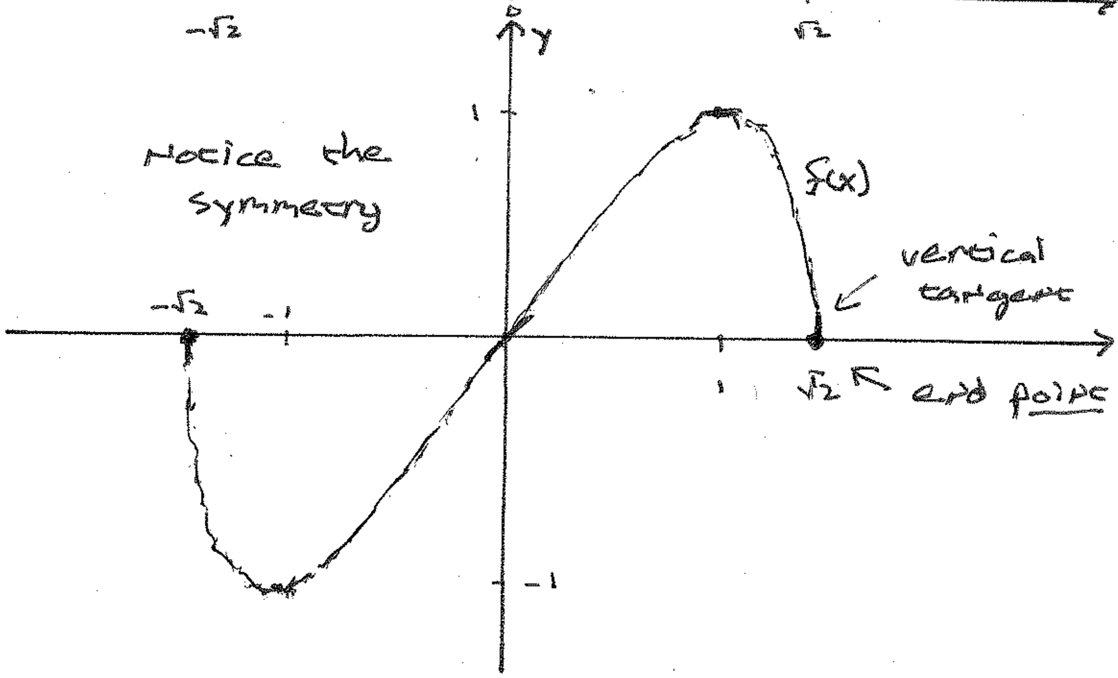
$= -\dfrac{2(x+1)(x-1)}{\sqrt{(\sqrt{2}+x)(\sqrt{2}-x)}}$

max @ $(1, 1)$

$f''(x) = \dfrac{2x(x^2-3)}{(2-x^2)^{3/2}}$

$= \dfrac{2x(x+\sqrt{3})(x-\sqrt{3})}{(2-x^2)\sqrt{(\sqrt{2}+x)(\sqrt{2}-x)}}$





ex 5: sketch $g(x) = e^{2/x}$

$$g'(x) = -\frac{2e^{2/x}}{x^2}$$

$$g''(x) = \frac{4e^{2/x}(x+1)}{x^4}$$

(C) end behavior.

$$\lim_{x\to\infty} e^{2/x} = e^{\lim_{x\to\infty}\frac{2}{x}} = e^0 = 1$$

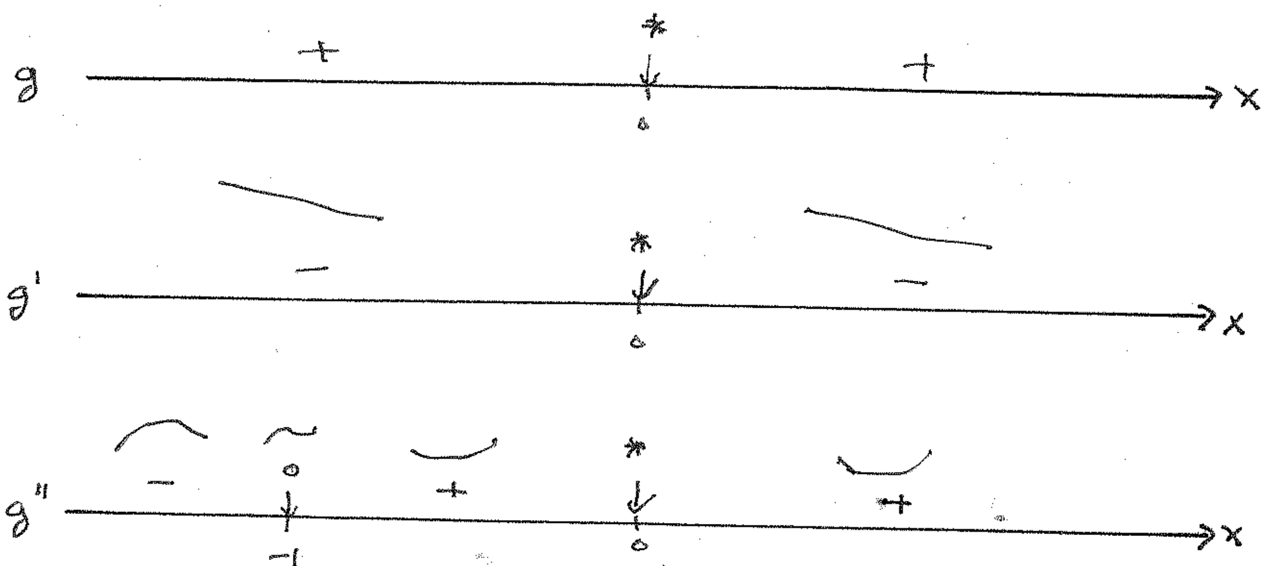
so we have a H.A. @ $y=1$.

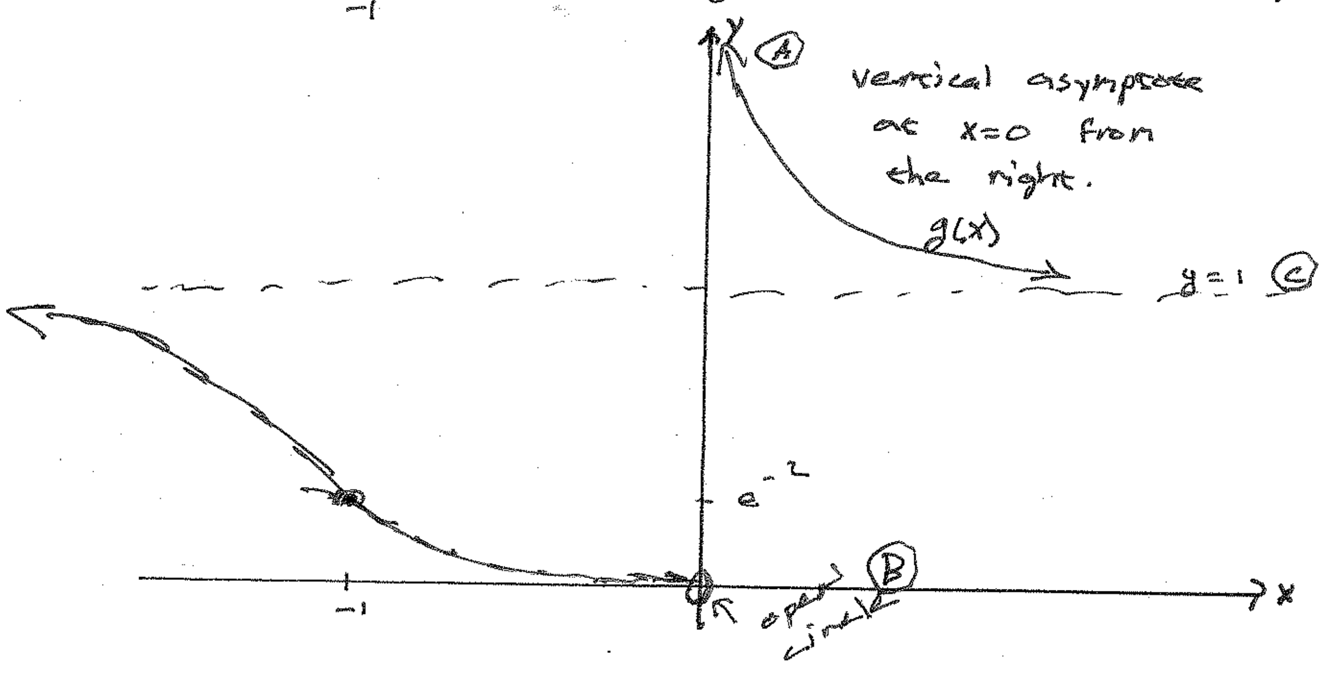
what happens at $x=0$?

(A) $$\lim_{x\to 0^+} e^{2/x} = e^{\lim_{x\to 0^+}\frac{2}{x}} = e^{\infty} = \infty$$

(B) $$\lim_{x\to 0^-} e^{2/x} = e^{\lim_{x\to 0^-}\frac{2}{x}} = e^{-\infty} = 0$$

poi $(-1, e^{-2})$





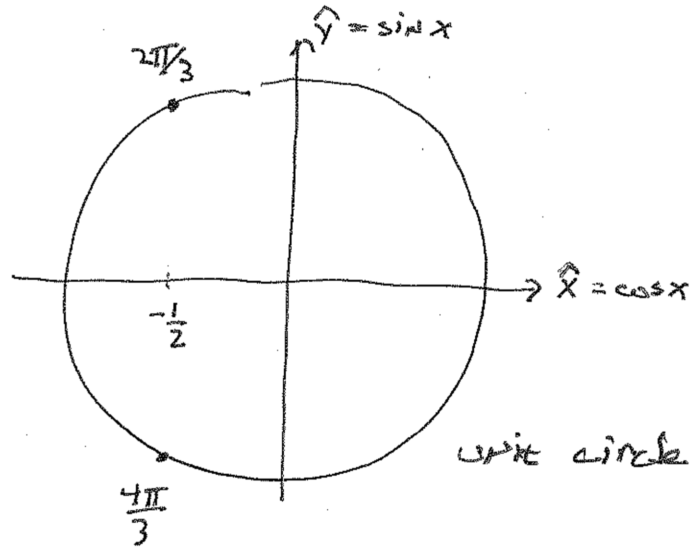
ex6: sketch $h(x) = \dfrac{\sin x}{2+\cos x}$ periodic on $[0, 2\pi]$

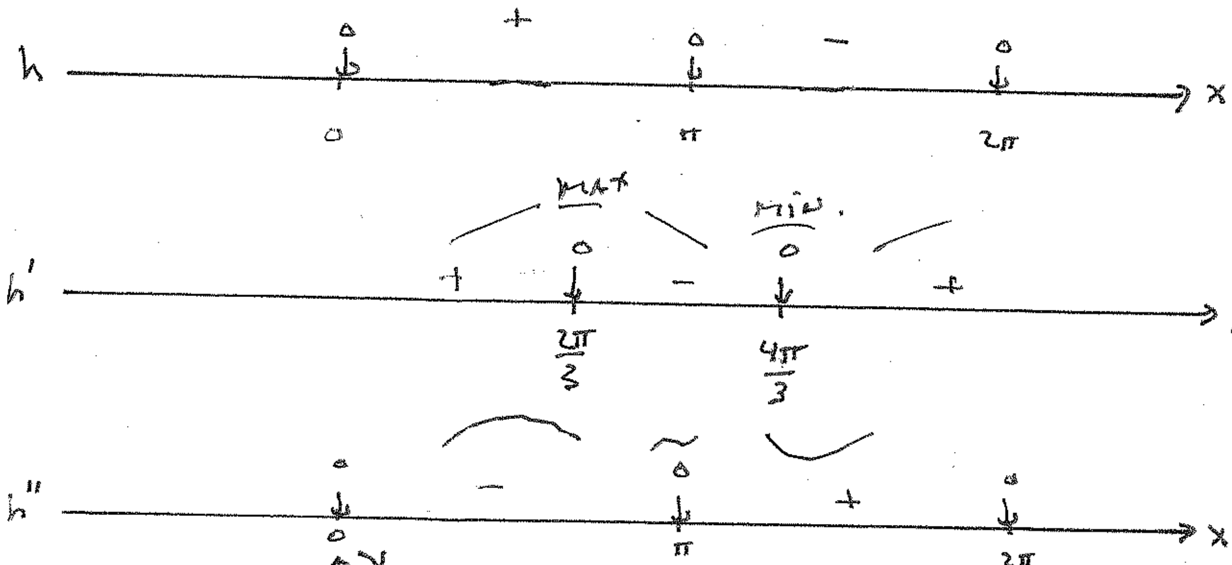
$$h'(x) = \frac{2\cos x + 1}{(2+\cos x)^2}$$

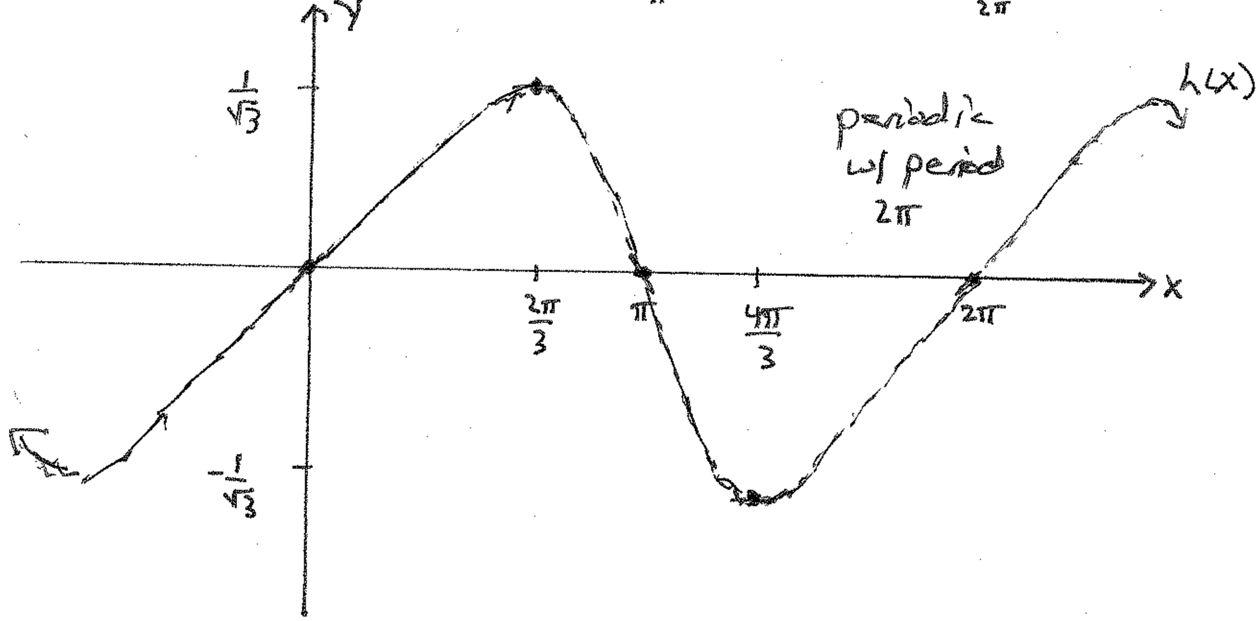
$$h''(x) = \frac{2\sin x(\cos x - 1)}{(2+\cos x)^3}$$

max $\left(\frac{2\pi}{3}, \frac{1}{\sqrt{3}}\right)$

min $\left(\frac{4\pi}{3}, -\frac{1}{\sqrt{3}}\right)$







ex: $f(x) = x^{2/3}(x^2 - 2x - 6)$

$= x^{2/3}(x-(1+\sqrt{7}))(x-(1-\sqrt{7}))$

$= x^{8/3} - 2x^{5/3} - 6x^{2/3}$

solve $0 = x^2 - 2x - 6$

$\Rightarrow x = \dfrac{2 \pm \sqrt{4-4(1)(-6)}}{2}$

$\Rightarrow x = \dfrac{2 \pm \sqrt{28}}{2}$

$\Rightarrow x = 1 \pm \sqrt{7}$

$f'(x) = \frac{8}{3}x^{5/3} - \frac{10}{3}x^{2/3} - \frac{12}{3}x^{-1/3}$

$= \frac{2}{3}x^{-1/3}(4x^{6/3} - 5x^{3/3} - 6)$

$= \frac{2}{3}x^{-1/3}(4x+3)(x-2)$

solve $0 = 4x^2 - 5x - 6$

$= 4x^2 - 8x + 3x - 6$

$= 4x(x-2) + 3(x-2$

$= (4x+3)(x-2)$

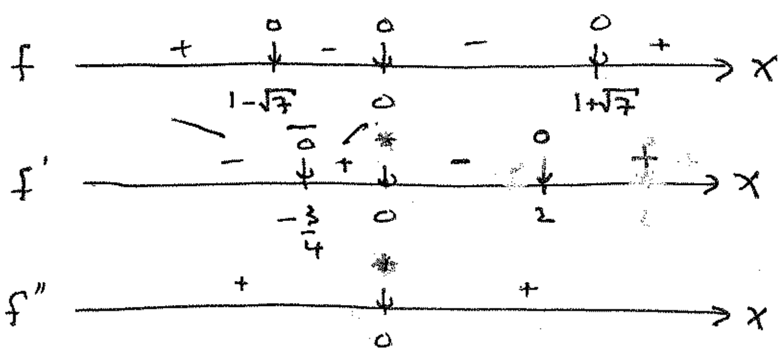
$f''(x) = \frac{40}{9}x^{2/3} - \frac{20}{9}x^{-1/3} + \frac{12}{9}x^{-4/3}$

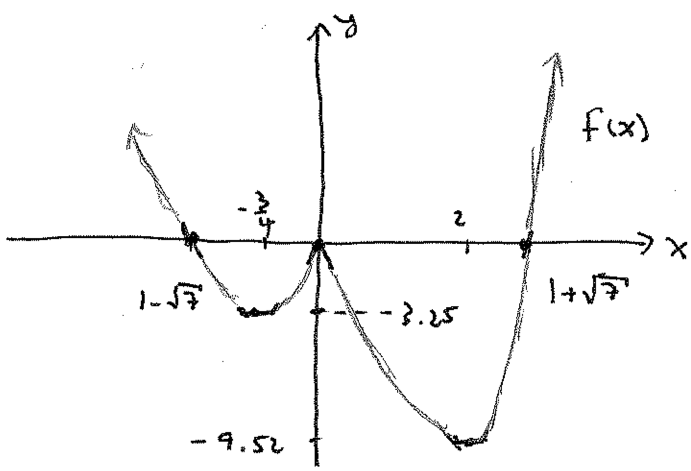
$= \frac{4}{9}x^{-4/3}(10x^{+6/3} - 5x^{3/3} + 3)$

solve $0 = 10x^2 - 5x + 3$

$\Rightarrow x = \dfrac{5 \pm \sqrt{25-4(10)(3)}}{2(10)}$

$\Rightarrow$ No real sol.





$f(1-\sqrt{7})$

$f(0)$

$f(1+\sqrt{7})$

$f(-\frac{3}{4}) \approx -3.25$

$f(2) \approx -9.52$