2.6: Limits @ Infinity.

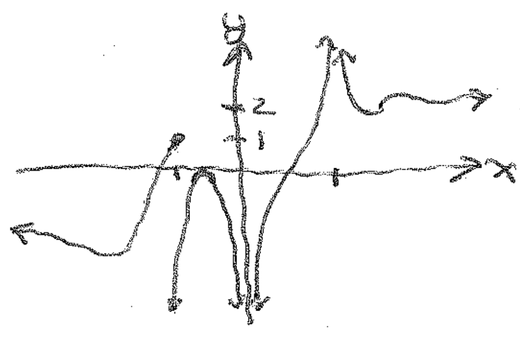
Ex1: Numerically explore $\lim_{x \to \infty} 3 + \frac{1}{x} \sin x$

Dfn: Let f be a fct defined on some interval (a, ∞). Then $\lim_{x \to \infty} f(x) = L$ means that values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.

more precisely: $\forall\, \varepsilon > 0\ \exists\, N \in \mathbb{R}$ s.t.
if $x > N$ then $|f(x) - L| < \varepsilon$

We call the line $y = L$ a horizontal asymptote of $y = f(x)$ if $\lim_{x \to -\infty} f(x) = L$ or $\lim_{x \to \infty} f(x) = L$.

Ex2: Find infinite limits, limits @ infinity, and the eqt. of hor. asy.



Important examples to know

$$\lim_{x \to \pm\infty} \frac{1}{x}$$

$$\lim_{x \to \pm\infty} \frac{1}{x^2}$$

$$\lim_{x \to \pm\infty} \arctan(x)$$

<u>Thm</u>: If $r > 0$ is a rational number, then

$$\lim_{x \to \pm\infty} \frac{1}{x^r} = 0$$

<u>Q</u>: Why a rational number?

<u>ex3</u>: $\displaystyle \lim_{x \to \infty} \frac{5x^3 + 2x - 7}{6x^3 + 3x^2 + 1}$ $\qquad$ $\displaystyle \lim_{x \to \infty} \left(\sqrt{4x^2 + 5x} - 2x\right)$

<u>ex4</u>: $\displaystyle \lim_{x \to -\infty} \left(x + \sqrt{x^2 + 2x}\right) \cdot \frac{\left(x - \sqrt{x^2 + 2x}\right)}{x - \sqrt{x^2 + 2x}}$

$$= \lim_{x \to -\infty} \frac{-2x}{x - \sqrt{x^2 + 2x}}$$

$$\cancel{= \lim_{x \to -\infty} \frac{-2}{1 - \sqrt{1 + \frac{2}{x}}} = -\infty}$$

$$= \lim_{x \to -\infty} \frac{-2x}{x - \sqrt{x^2\left(1 + \frac{2}{x}\right)}}$$

$$= \lim_{x \to -\infty} \frac{-2x}{x - |x|\sqrt{1 + \frac{2}{x}}}$$

$$= \lim_{x \to -\infty} \frac{-2}{1 + \sqrt{1 + \frac{2}{x}}}$$

$$= -1$$

More important examples

$$\lim_{x \to \pm\infty} e^x$$

$$\lim_{x \to \pm\infty} x^2$$

$$\lim_{x \to \pm\infty} x^3$$

ex 5: $$\lim_{x \to \infty} \frac{x^3 - 2x + 3}{5 - 2x^2}$$

ex6: Find a value of N s.t. $x > N \Rightarrow |f(x) - L| < \varepsilon$

for ~~[illegible]~~ $\lim_{x \to \infty} \frac{\sqrt{4x^2+1}}{x+1} = 2$ and $\varepsilon = 0.1$.

By graphing: ~~[illegible]~~ N is @ least 18.86752.