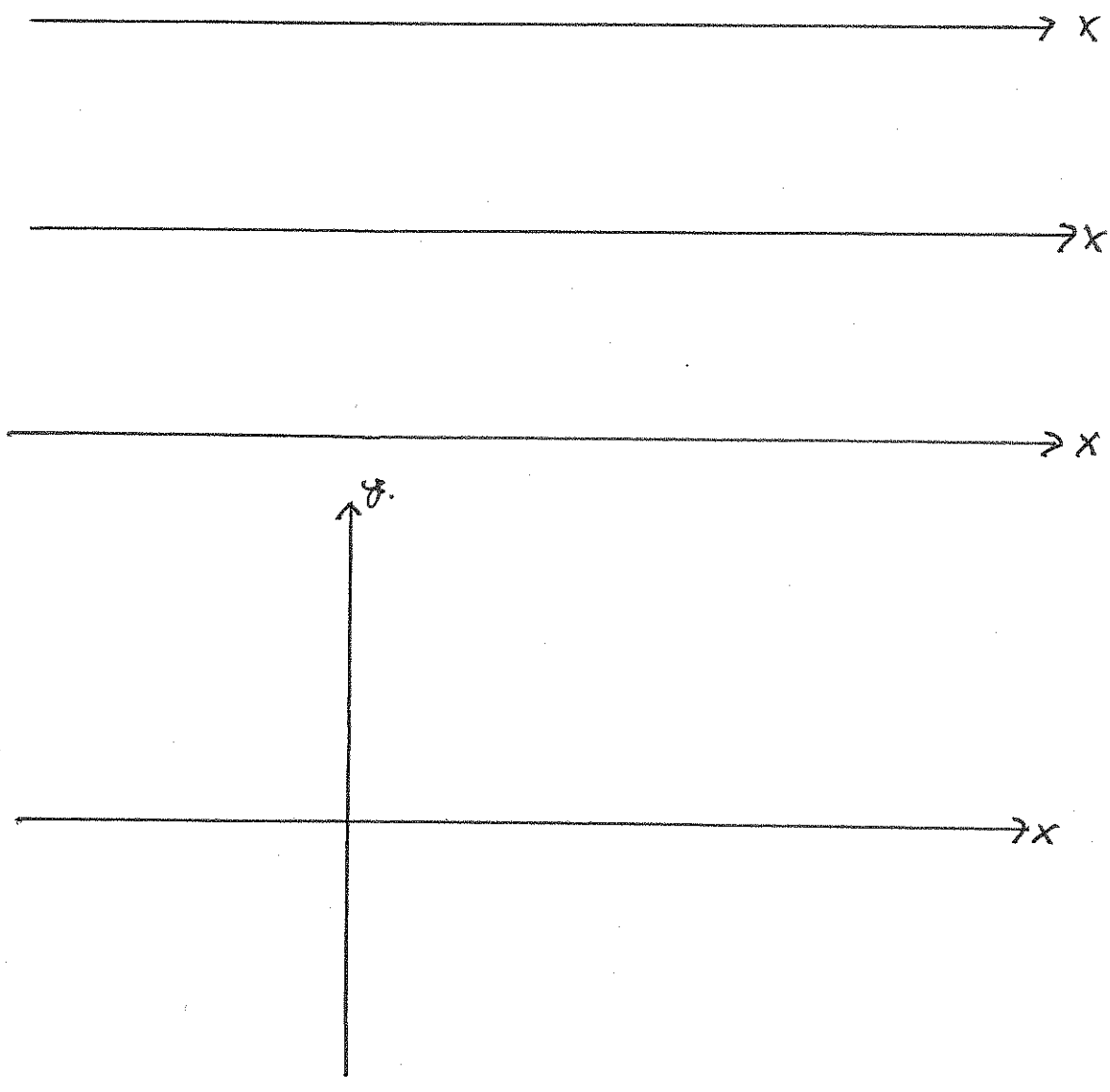
4.5: Curve Sketching

ex1: sketch $f(x) = x^4 - 4x^3 + 10$

$$f'(x) = 4x^3 - 12x^2$$
$$= 4x^2(x-3)$$

$$f''(x) = 12x^2 - 24x$$
$$= 12x(x-2)$$



ex 2: sketch $g(x) = x^3 - 27x$

$= x(x^2 - 27)$

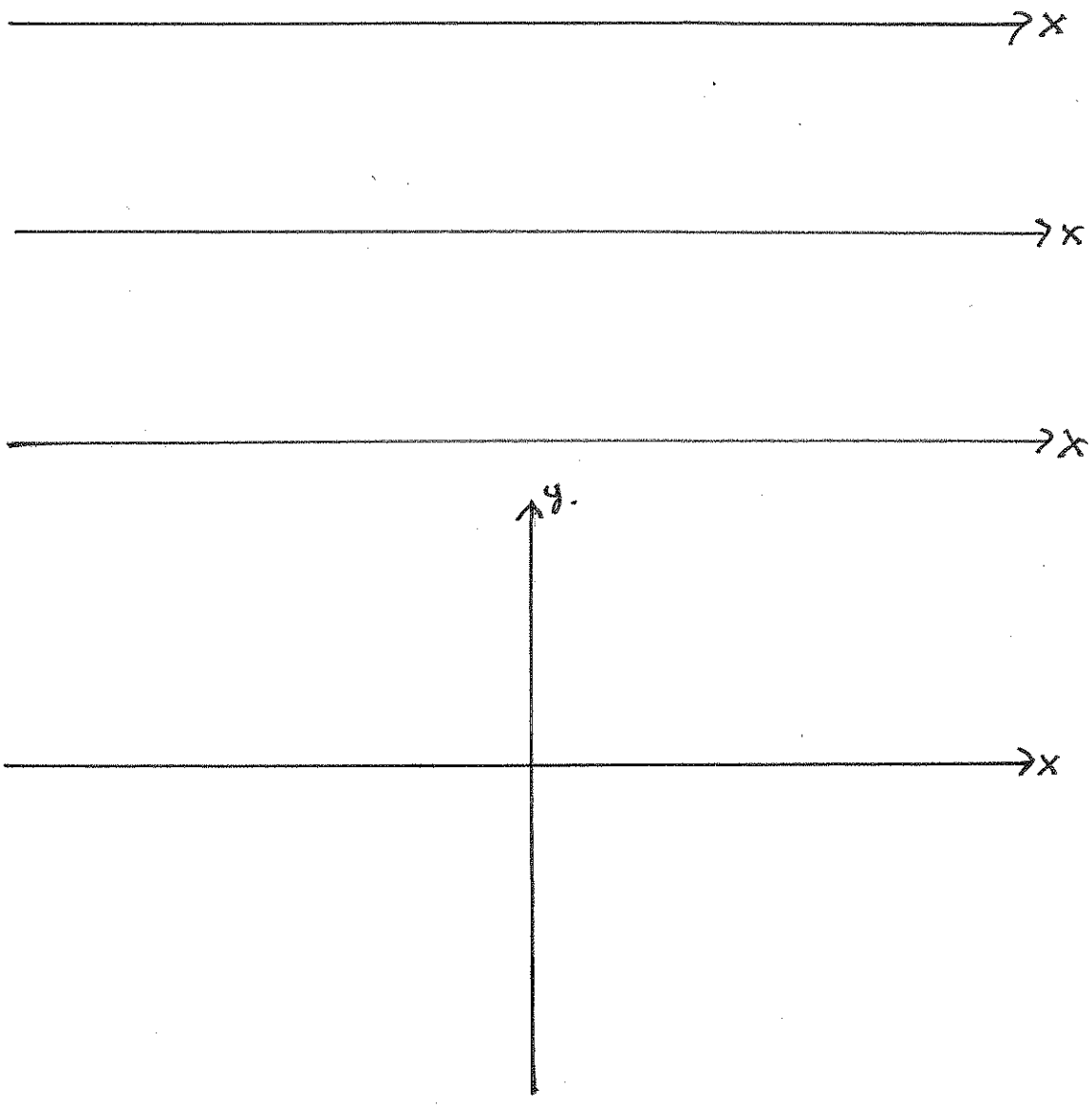
$= x(x + \sqrt{27})(x - \sqrt{27})$

$g'(x) = 3x^2 - 27$

$= 3(x^2 - 9)$

$= 3(x+3)(x-3)$

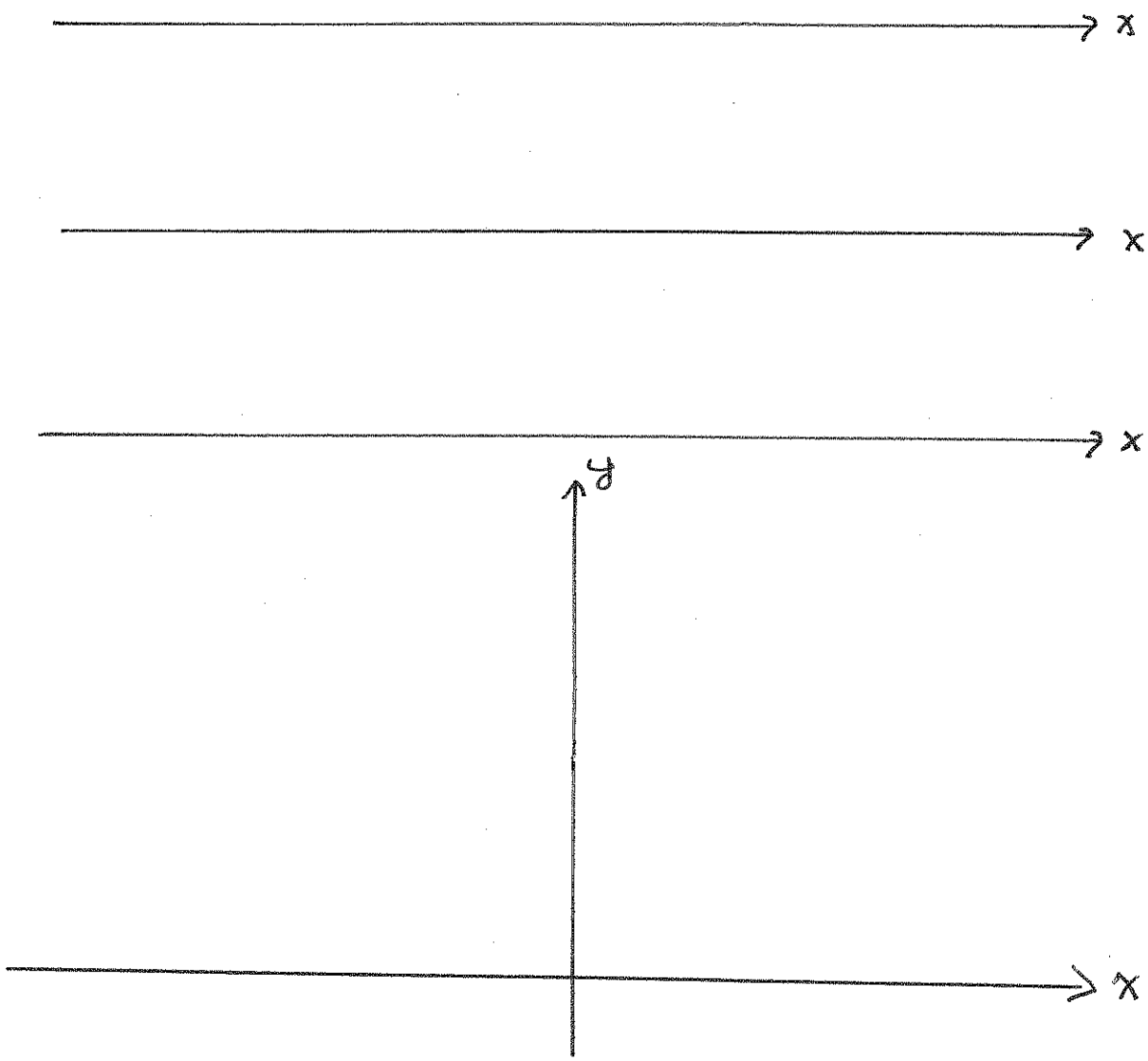
$g''(x) = 6x$



ex 3: sketch $h(x) = \dfrac{(x+1)^2}{1+x^2}$

$$h'(x) = \frac{-2(x+1)(x-1)}{(1+x^2)^2}$$

$$h''(x) = \frac{4x(x+\sqrt{3})(x-\sqrt{3})}{(1+x^2)^3}$$



ex4: sketch $f(x) = x\sqrt{2-x^2}$

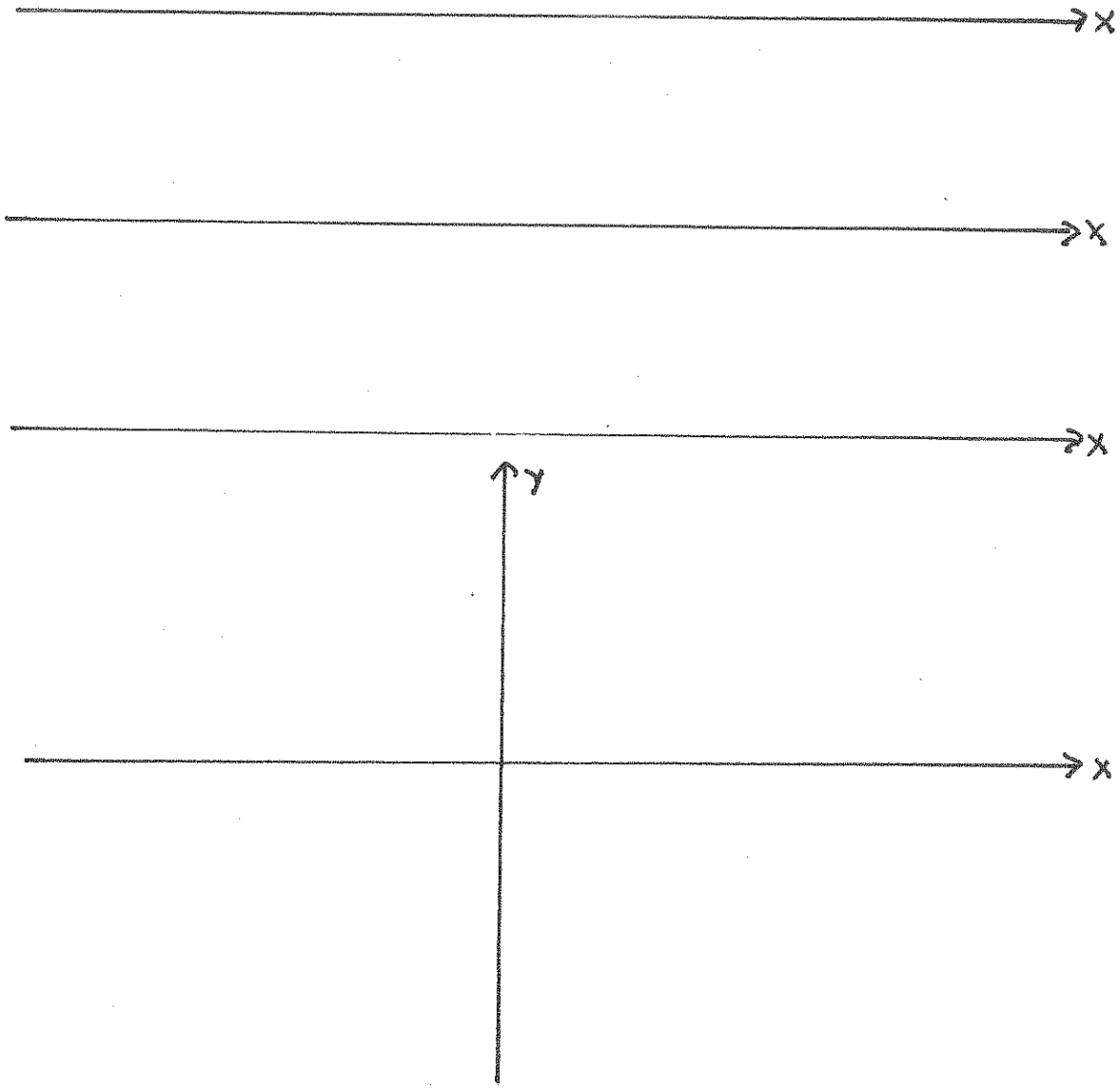
$$f(x) = x\sqrt{(\sqrt{2}+x)(\sqrt{2}-x)}$$

$$f'(x) = -\frac{2(x^2-1)}{\sqrt{2-x^2}}$$

$$= -\frac{2(x+1)(x-1)}{\sqrt{(\sqrt{2}+x)(\sqrt{2}-x)}}$$

$$f''(x) = \frac{2x(x^2-3)}{(2-x^2)^{3/2}}$$

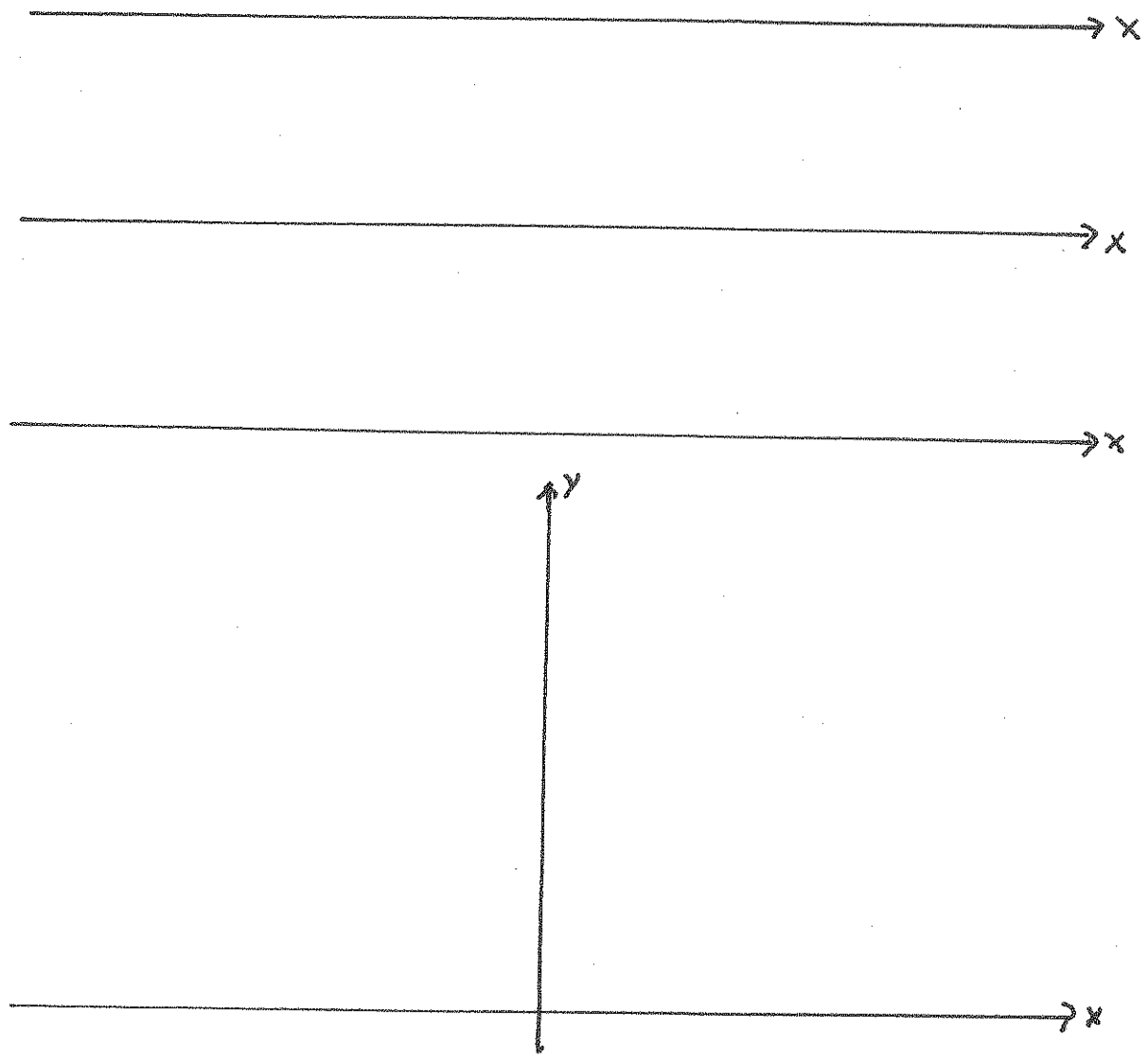
$$= \frac{2x(x+\sqrt{3})(x-\sqrt{3})}{(2-x^2)\sqrt{(\sqrt{2}+x)(\sqrt{2}-x)}}$$



ex 5: sketch $g(x) = e^{2/x}$

$$g'(x) = -\frac{2e^{2/x}}{x^2}$$

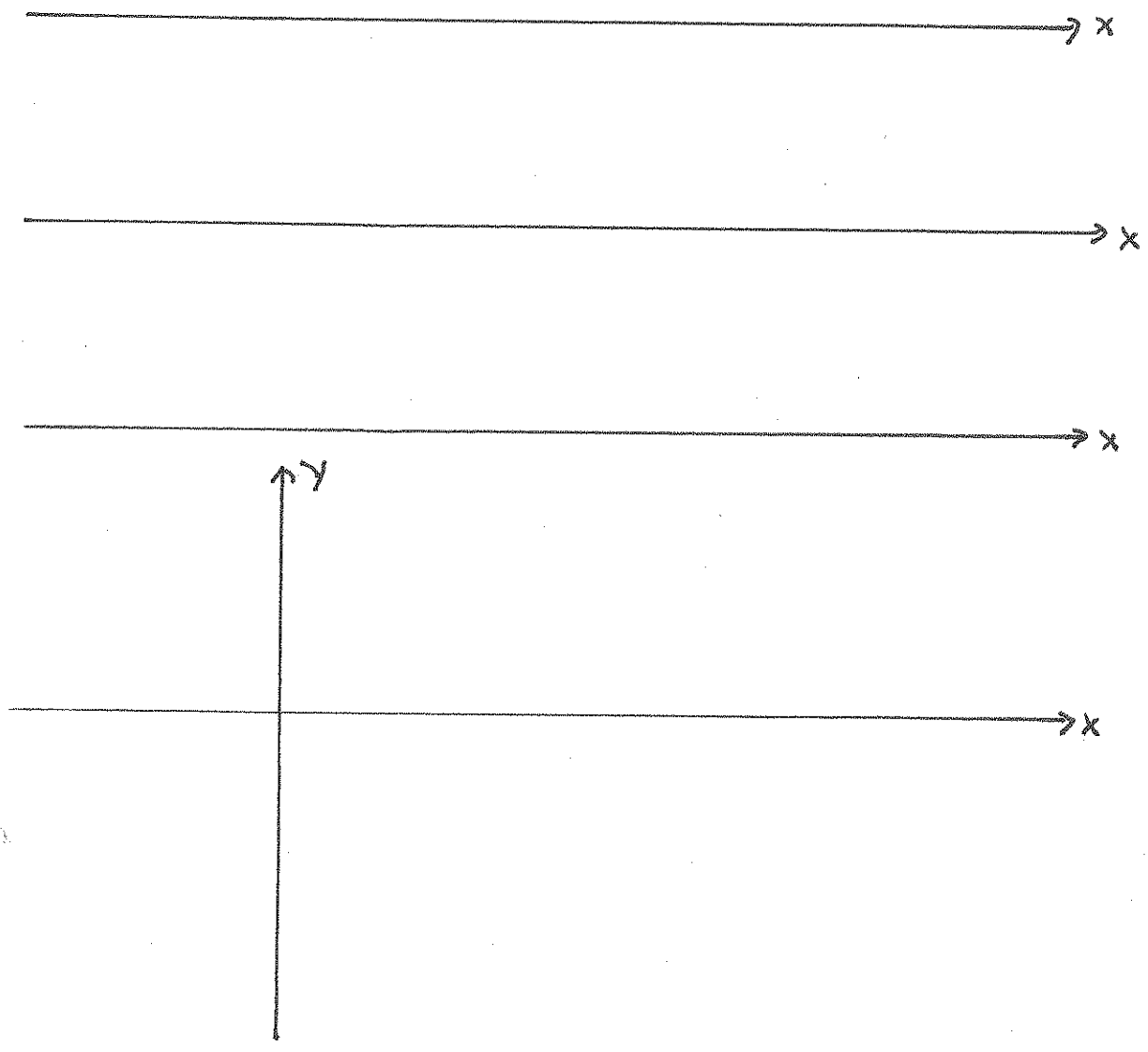
$$g''(x) = \frac{4e^{2/x}(x+1)}{x^4}$$



ex 6: sketch $h(x) = \dfrac{\sin x}{2 + \cos x}$

$$h'(x) = \frac{2\cos x + 1}{(2 + \cos x)^2}$$

$$h''(x) = \frac{2\sin x(\cos x - 1)}{(2 + \cos x)^3}$$



ex 7: sketch $f(x) = x^{2/3}(x^2 - 2x - 6)$

$$f'(x) = \frac{2}{3}x^{-1/3}(4x+3)(x-2)$$

$$f''(x) = \frac{4}{9}x^{-4/3}(10x^2 - 5x + 3)$$