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Math 291

Prove that $\lim_{x \to 4} x^2 - 3x = 4$ by δ, ε. Let $\varepsilon > 0$ be given.
Let's show that given any epsilon, we can find a delta.

$\rightarrow$ If $0 < |x-4| < \delta$, then we want $|\underbrace{x^2-3x-4}| < \varepsilon$

$\rightarrow |(x-4)(x+1)| < \varepsilon$

$\rightarrow |x-4| \, |\underbrace{x+1}| < \varepsilon$

What is the largest constant C I can replace this with?

So let's assume a small delta, such that $|x-4| < 1$, so $3 < x < 5$, and $4 < x+1 < 6$. 6 will be a reasonable constant to choose.

$\rightarrow$ Now, if $|x-4| < \delta$, then we want $6\underbrace{|x-4|} < \varepsilon$

$\rightarrow$ so $|x-4| < \varepsilon/6$

So there are two restrictions on delta:

$|x-4| < 1$ and $|x-4| < \varepsilon/6$

So given an epsilon, we must choose a delta that is at the most 1 (whichever is smaller between 1 and epsilon given - $\varepsilon/6$)

So, given $\varepsilon > 0$, let $\delta = \min\{1, \varepsilon/6\}$

Now I have shown that given any epsilon, we can find a delta.