claim: $\mathbb{R}$ is uncountable

□ proof.

Assume the real numbers on $0 < x < 1$ are countable.

$\Rightarrow$ there is a 1-1 correspondence between $\mathbb{N}$ and the reals on $(0, 1)$

$\Rightarrow$ we can write the n^{th} real as

$a_n = 0.d_1 d_2 d_3, \ldots, d_n, \ldots.$

notice that d_n is either odd or even.

Let's consider the real number $x = b_1 b_2 b_3 \ldots$

where $b_i = \begin{cases} 0, & i^{th} \text{ digit of the } i^{th} \text{ real is odd.} \\ 1, & i^{th} \text{ digit of the } i^{th} \text{ real is even.} \end{cases}$

$\Rightarrow$ $x \neq a_i$ for $i = 1, 2, 3, \ldots$

$\Rightarrow$ there is a real number on $(0, 1)$ that isn't in a 1-1 correspondence w/ $\mathbb{N}$.

$\Rightarrow$ the reals on $(0, 1)$ are not countable $\Rightarrow\Leftarrow$

Since $(0,1) \subset \mathbb{R}$ and $(0,1)$ isn't countable, we have shown $\mathbb{R}$ is uncountable. ■