Prove: $1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

Proof by Induction

Step 1: show claim is true for $n=1$.

$$1^2 \stackrel{?}{=} \frac{1\cdot 2\cdot(2\cdot 1+1)}{6} = \frac{1\cdot 2\cdot 3}{6} = 1. \quad \underline{\text{true}}.$$

Step 2: assume result is true for $n=k$, and show this implies it is true for $n=k+1$.

* So assume $1^2+2^2+\dots+k^2 = \frac{k(k+1)(2k+1)}{6}$.

Now show result must hold for $n=k+1$, ie, that

** $1^2+2^2+\dots+k^2+(k+1)^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$.

So with the inductive hypothesis * in mind, we try to show ** is true. So:

$$\underbrace{1^2+2^2+\dots+k^2}+(k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{(k+1)(2k^2+k)}{6} + (k+1)^2 = \frac{(k+1)(2k^2+k)}{6} + \frac{6(k+1)^2}{6}$$

$$= \frac{(k+1)}{6}(2k^2+k+6k+6) = \frac{(k+1)}{6}(2k^2+7k+6)$$

$$= \frac{(k+1)}{6}(k+2)(2k+3) = \frac{(k+1)(k+2)(2k+3)}{6}.$$