<u>claim</u>: Any positive integer is div. by 3 iff the sum of its digits are div. by 3.

<u>proof.</u>

Let the positive integer k w/ $N+1$ digits be given.

$\Rightarrow$ k is of the form $a_N a_{N-1} \dots a_2 a_1 a_0$

$\Rightarrow$ $k = a_N \cdot 10^N + a_{N-1} \cdot 10^{N-1} + \dots + a_2 10^2 + a_1 \cdot 10 + a_0$

$= a_N(10^N - 1 + 1) + \dots + a_2(10^2 - 1 + 1) + a_1(10 - 1 + 1) + a_0$

$= \underbrace{99\dots9}_{N\ 9\text{'s}} a_N + \dots + 99a_2 + 9a_1 + (a_N + a_{N-1} + \dots + a_2 + a_1 + a_0)$

consider $\frac{k}{3} = \frac{99\dots9a_N + \dots + 9a_1 + (a_N + \dots + a_1 + a_0)}{3}$

$= 33\dots3a_N + \dots + 3a_1 + \frac{(a_N + \dots + a_1 + a_0)}{3}$

Hence k is divisible by 3 iff the sum of its digits is divisible by 3. ∎

Prove: The number of primes is infinite.

Method: Proof by contradiction. We will assume there are exactly n primes $p_1, \ldots, p_n$ and shows this gives a contradiction

<u>Proof</u>: Assume $p_1, \ldots, p_n$ are the only prime numbers.

Note that if a prime p_k divides $(a+b)$ and p_k divides a, it must also divide b.

Consider the number $(p_1 p_2 \ldots p_n) + 1$. If this number is also prime, we are done (we have found another prime, which contradicts our assumption).
If it is not prime, ie, it is composite, then it must be divisable by one of the primes $p_1, \ldots, p_n$, say p_k.

But if p_k divides $(p_1 \cdots p_n) + 1$, and it also must clearly divide $p_1 \cdots p_n$, then p_k must also divide 1, and no prime can divide 1.

So p_k cannot divide $(p_1 \cdots p_n) + 1$, ie, no prime can divide it, so it must be prime, which contradicts our original assumption. So the original assumption must be false. $\blacksquare$