4.1: Intro to Linear Spaces.

Dfn: V is a linear space (vector space) if $\forall\, f, g \in V$ and $c, k \in \mathbb{R}$

(1) $(f+g)+h = f+(g+h)$

(2) $f+g = g+f$

(3) $\exists$ a neutral element $n \in V$ s.t. $f+n = f$ $\forall\, f \in V$.

n is unique and denoted by 0

(4) $\forall\, f \in V$ $\exists\, g \in V$ s.t. $f+g=0$. This g is unique & denoted by $(-f)$

(5) $k(f+g) = kf + kg$

(6) $(c+k)f = cf + kf$

(7) $c(kf) = (ck)f$

(8) $1f = f$.

Key: A linear space is a set over which addition and scalar mult. are defined. Zero must be defined.

examples

$\mathbb{R}^n$

$F(\mathbb{R}, \mathbb{R})$ (set of fcts from $\mathbb{R}$ to $\mathbb{R}$)

$\mathbb{R}^{n \times m}$ matrices.

$\mathbb{R}^\infty$

$\mathbb{C}$

Dfn: A subset W of a linear space V is called a subspace of V if.

(a) W contains the neutral element 0 of V.

(b) W is closed under addition.

(c) W is closed under scalar mult.

which are subspaces.

Ex: Diagonal 3×3 matrices

Ex: 3×3 matrices w/ non-neg. entries

Ex: 3×3 matrices in RREF form.

Ex: The geometric sequences.

Dfn: Span, LI, Basis, & coords in a linear space.

thm: If a lin. space V has a basis w/ n elements, then all other bases of V consist of n elements as well. We say $\dim(V) = n$.

To find a basis...

(a) write a typical element in terms of arb. consts.

(b) using the arb. constants as coefficients, express your typical element as a lin comb. of some elements of V. (SPAN)

(c) verify that these elements of V are L.I.

... if so, it is a basis.

$\boxed{\begin{matrix} 4.1 \\ 3/3 \end{matrix}}$

Find a basis & determine dim.

ex: The space of all diag 2×2 matrices

ex: The space of all polys $f(x)$ in P_2 s.t. $f(1)=0$.

ex: The space of all matrices s.t. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} S = S \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$