6.1 : Determinants

The 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible if $\det A = ad - bc \neq 0$.

More generally, the $n\times n$ matrix A is invertible if $\det A \neq 0$. ... But how do you find $\det A$?

In the 3×3 case, we need the $\operatorname{im}(A) = \mathbb{R}^3$ or the cols to be L.I. One way to determine this is as follows. Let $A = [\vec{u}\ \vec{v}\ \vec{w}]$

$$\det A = \vec{u} \cdot (\vec{v} \times \vec{w}).$$

Q: why?

ex1: calculate $\det A$ for $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & -3 \\ 4 & 5 & 1 \end{bmatrix}$

using dfn & Sarrus Rule.

Det. of a triangular or diag. matrix is simply the product of the diag ex2: Find the det A for $A = \begin{bmatrix} 1-\lambda & 1 & 1 \\ 0 & 3-\lambda & 3 \\ -2 & 1 & 1-\lambda \end{bmatrix}$.

what does this mean? key: $\lambda = 0, 2, 3$

Note: In the 3×3 case, $\det B = -\det A$ if A & B differ only in a single col. swap.

evaluating determinates ... the signs.

ex3: $A = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 1 & 3 \\ 1 & 4 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 3 \\ 1 & 1 & 0 & 1 \\ 1 & 4 & 2 & 2 \end{bmatrix}$ row & col swaps.

$C = \begin{bmatrix} 0 & 4 & 1 & 3 \\ 0 & 2 & 2 & 1 \\ 1 & 3 & 1 & 2 \\ 2 & 2 & 1 & 4 \end{bmatrix}$