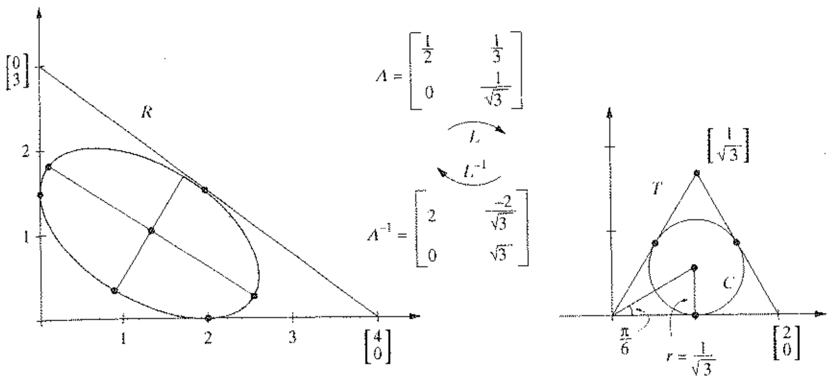

Figure 6.9: for Problem 6.3.48 and Problem 6.3.49.

$\sqrt{M} = \sqrt{\frac{25+\sqrt{193}}{18}} \approx 1.47$, and for the semi-minor axis we get

$\sqrt{m} = \sqrt{\frac{25-\sqrt{193}}{18}} \approx 0.79$.

True or False

Ch 6.TF.1 T, by Definition 6.1.1

Ch 6.TF.2 F; We have $\det(4A) = 4^4 \det(A)$, by Theorem 6.2.3a.

Ch 6.TF.3 F; Let $A = B = I_5$, for example

Ch 6.TF.4 T; We have $\det(-A) = (-1)^6 \det(A) = \det(A)$, by Theorem 6.2.3a.

Ch 6.TF.5 F; In fact, $\det(A) = 0$, since A fails to be invertible

Ch 6.TF.6 F; The matrix A fails to be invertible if $\det(A) = 0$ by Theorem 6.2.4.

Ch 6.TF.7 T, by Theorem 6.2.3a, applied to the columns.

Ch 6.TF.8 T, by Theorem 6.2.6.

Ch 6.TF.9 T, By theorem 6.1.4, a diagonal matrix is triangular as well.

Ch 6.TF.10 T, by Theorem 6.2.3b.

Ch 6.TF.11 T. Without computing its exact value, we will show that the determinant is positive. The pattern that contains all the entries 100 has a product of $100^4 = 10^8$, with two inversions. Each of the other $4! - 1 = 23$ patterns contains at most two entries 100, with the other entries being less than 10, so that the product of each of these patterns is less than $100^2 \cdot 10^2 = 10^6$. Thus the determinant in more than $10^8 - 23 \cdot 10^6 > 0$, so that the matrix in invertible.

Ch 6.TF.**12** F; The correct formula is $\det(A^{-1}) = \frac{1}{\det(A^T)}$, by Theorems 6.2.1 and 6.2.8.

Ch 6.TF.**13** T; The matrix A is invertible.

Ch 6.TF.**14** T; Any nonzero noninvertible matrix A will do.

Ch 6.TF.**15** T, by Theorem 6.2.7.

Ch 6.TF.**16** F, by Theorem 6.3.1. The determinant can be -1.

Ch 6.TF.**17** T, by Theorem 6.2.6.

Ch 6.TF.**18** F; The second and the fourth column are linearly dependent.

Ch 6.TF.**19** T; The determinant is 0 for $k = -1$ or $k = -2$, so that the matrix is invertible for all *positive* k.

Ch 6.TF.**20** F. There is only one pattern with a nonzero product, containing all the 1's. Since there are three inversions in this pattern, $\det A = -1$.

Ch 6.TF.**21** T; Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$. The column vectors of A are orthogonal and they all have length 2.

Ch 6.TF.**22** F; Let $A = \begin{bmatrix} 8 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, for example.

Ch 6.TF.**23** F; In fact, $\det(A) = \det[\vec{u}\ \vec{v}\ \vec{w}] = -\det[\vec{v}\ \vec{u}\ \vec{w}] = -\vec{v} \cdot (\vec{u} \times \vec{w})$. We have used Theorem 6.2.3b and Definition 6.1.1.

Ch 6.TF.**24** T; Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, for example.

Ch 6.TF.**25** F; Note that $\det \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = 2$.

Ch 6.TF.**26** T, by Theorem 6.3.9.

Ch 6.TF.**27** T, by Theorem 6.3.3, since $\|\vec{v}_i^{\perp}\| \leq \|\vec{v}_i\| = 1$ for all column vectors $\vec{v}_i$.

Ch 6.TF.**28** T; We have $\det(A) = \det(\text{rref } A) = 0$.

Ch 6.TF.**29** F; Let $A = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$, for example. See Theorem 6.2.10.

Ch 6.TF.**30** F; Let $A = 2I_2$, for example

Ch 6.TF.31 F; Note that $\det(S^{-1}AS) = \det(A)$ but $\det(2A) = 2^3(\det A) = 8(\det A)$.

Ch 6.TF.32 F; Note that $\det(S^TAS) = (\det S)^2(\det A)$ and $\det(-A) = -(\det A)$ have opposite signs.

Ch 6.TF.33 F; Let $A = 2I_2$, for example.

Ch 6.TF.34 F; Let $A = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, for example.

Ch 6.TF.35 F; Let $A = I_2$ and $B = -I_2$, for example.

Ch 6.TF.36 T; Note that $\det(B) = -\det(A) < \det(A)$, so that $\det(A) > 0$.

Ch 6.TF.37 T; Let's do Laplace expansion along the first row, for example (see Theorem 6.2.10).

Then $\det(A) = \sum_{j=1}^{n}(-1)^{1+j}a_{1j}\det(A_{1j}) \neq 0$. Thus $\det(A_{1j}) \neq 0$ for at least one j, so that A_{1j} is invertible.

Ch 6.TF.38 T; Note that $\det(A)$ and $\det(A^{-1})$ are both integers, and $(\det A)(\det A^{-1}) = 1$. This leaves only the possibilities $\det(A) = \det(A^{-1}) = 1$ and $\det(A) = \det(A^{-1}) = -1$.

Ch 6.TF.39 T, since $\text{adj}(A) = (\det A)(A^{-1})$, by Theorem 6.3.9.

Ch 6.TF.40 F; Note that $\det(A^2) = (\det A)^2$ cannot be negative, but $\det(-I_3) = -1$.

Ch 6.TF.41 T; The product associated with the diagonal pattern is odd, while the products associated with all other patterns are even. Thus the determinant of A is odd, so that A is invertible, as claimed.

Ch 6.TF.42 F; Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 5 & 2 \end{bmatrix}$, for example

Ch 6.TF.43 T; Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $a \neq 0$, let $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$; if $b \neq 0$, let $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$; if $c \neq 0$, let $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, and if $d \neq 0$, let $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

Ch 6.TF.44 T; Use Gaussian elimination for the first column only to transform A into a matrix of the form

$$B = \begin{bmatrix} 1 & \pm1 & \pm1 & \pm1 \\ 0 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{bmatrix}$$

Note that $\det(B) = \det(A)$ or $\det(B) = -(\det A)$. The stars in matrix B all represent numbers $(\pm1) \pm (\pm1)$, so that they are 2, 0, or -2. Thus the determinant of the 3×3 matrix M containing the stars is divisible by 8, since each of the 6 terms in Sarrus' rule is 8, 0 or -8. Now perform Laplace expansion down the first column of B to see that $\det(M) = \det(B) = +/-\det(A)$.

Ch 6.TF.**45** T; $A(\text{adj}A) = A(\det(A)A^{-1}) = \det(A)I_n = \det(A)A^{-1}A = \text{adj}(A)A$.

Ch 6.TF.**46** T; Laplace expansion along the second row gives $\det(A) = -k\det\begin{bmatrix}1 & 2 & 4\\ 8 & 9 & 7\\ 0 & 0 & 5\end{bmatrix} + C = 35k + C$, for some constant C (we need not compute that $C = -259$). Thus A is invertible except for $k = \frac{-C}{35}$ (which turns out to be $\frac{259}{35} = \frac{37}{5} = 7.4$).

Ch 6.TF.**47** F; $A = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}$ and $B = \begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$ are both orthogonal and $\det(A) = \det(B) = 1$. However, $AB \neq BA$.