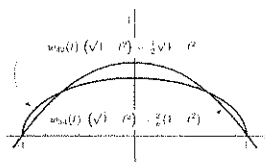


b. For $f(t) = \sqrt{1-t^2}$ we have $\|f\|_{32} = \sqrt{\frac{1}{2} \int_{-1}^1 (1-t^2) dt} = \sqrt{2/3}$ and $\|f\|_{34} = \sqrt{\langle \sqrt{1-t^2}, \sqrt{1-t^2} \rangle_{34}} = \sqrt{\langle 1, 1-t^2 \rangle_{34}} = \sqrt{1-1/4} = \sqrt{3/4}$.



True or False

Ch 5.TF.1 T, since $(A+B)^T = A^T + B^T = A+B$

Ch 5.TF.2 T, by Theorem 5.3.4

Ch 5.TF.3 F. Consider $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

Ch 5.TF.4 T. First note that $A^T = A^{-1}$, by Theorem 2.4.8. Thus A is orthogonal, by Theorem 5.3.7.

Ch 5.TF.5 F. The correct formula is $\text{proj}_L(\vec{x}) = (\vec{x} \cdot \vec{u})\vec{u}$, by Definition 2.2.1.

Ch 5.TF.6 T, since $(7A)^T = 7A^T = 7A$.

Ch 5.TF.7 F. Consider $T(\vec{x}) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \vec{x}$.

Ch 5.TF.8 T, by Theorem 5.3.9.b

Ch 5.TF.9 T, by Theorem 5.3.4a

Ch 5.TF.10 F. We have $(AB)^T = B^T A^T$, by Theorem 5.3.9a.

Ch 5.TF.11 T. If A is orthogonal, then $A^T = A^{-1}$, and A^{-1} is orthogonal by Theorem 5.3.4b.

Ch 5.TF.12 F. Consider $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

Ch 5.TF.13 F. Consider $A = B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. Then $AB^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ isn't equal to $B^T A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

Ch 5.TF.14 F. It is required that the columns of A be orthonormal (Theorem 5.3.10). As a counterexample, consider

$$A = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \text{ with } AA^T = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}.$$

Ch 5.TF.15 T, since $(ABBA)^T = A^T B^T B^T A^T = ABBA$, by Theorem 5.3.9a

Ch 5.TF.16 T, since $A^T B^T = (BA)^T = (AB)^T = B^T A^T$, by Theorem 5.3.9a

Ch 5.TF.17 F. $\dim(V) + \dim(V^\perp) = 5$, by Theorem 5.1.8c. Thus one of the dimensions is even and the other odd.

Ch 5.TF.18 T. Consider the QR factorization (Theorem 5.2.2)

Ch 5.TF.19 F. The Pythagorean Theorem holds for orthogonal vectors \vec{x}, \vec{y} only (Theorem 5.1.9)

Ch 5.TF.20 T. $\det \begin{bmatrix} a & c \\ b & d \end{bmatrix} = ad - bc = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Ch 5.TF.21 F. As a counterexample, consider $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

Ch 5.TF.22 T, by Theorem 5.4.1.

Ch 5.TF.23 T, by Theorem 5.4.2a.

Ch 5.TF.24 F. Consider $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, or any other symmetric matrix that fails to be orthogonal.

Ch 5.TF.25 F. $\det \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = -1 - 0 = -1$, yet $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ is orthogonal.

Ch 5.TF.26 T. $[\frac{1}{2}(A - A^T)]^T = \frac{1}{2}(A - A^T)^T = \frac{1}{2}(A^T - A) = -[\frac{1}{2}(A - A^T)]$.

Ch 5.TF.27 T, since the columns are unit vectors.

Ch 5.TF.28 T. Use the Gram-Schmidt process to construct such a basis (Theorem 5.2.1)

Ch 5.TF.29 F. The columns fail to be unit vectors (use Theorem 5.3.3b)

Ch 5.TF.30 T, by definition of an orthogonal projection (Theorem 5.1.4).

Ch 5.TF.31 T. Try $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, so that $A + B = \begin{bmatrix} 1 + \cos \theta & -\sin \theta \\ \sin \theta & 1 + \cos \theta \end{bmatrix}$. It is required that $\begin{bmatrix} 1 + \cos \theta \\ \sin \theta \end{bmatrix}$ and $\begin{bmatrix} -\sin \theta \\ 1 + \cos \theta \end{bmatrix}$ be unit vectors, meaning that $1 + 2 \cos \theta + \cos^2 \theta + \sin^2 \theta = 2 + 2 \cos \theta = 1$, or $\cos \theta = -\frac{1}{2}$, and $\sin \theta = \pm \frac{\sqrt{3}}{2}$. Thus $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$ is a solution.

Ch 5.TF.32 F. Consider $A = \begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$, for example, representing a rotation combined with a scaling.

Ch 5.TF.33 F. Consider $A = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$.

Ch 5.TF.34 T. By Definition 5.1.12, quantity $\cos(\theta) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|}$ is positive, so that θ is an acute angle.

Ch 5.TF.35 T. In Theorem 5.4.1, let $A = B^T$ to see that $(\text{im}(B^T))^\perp = \ker(B)$. Now take the orthogonal complements of both sides and use Theorem 5.1.8d.

Ch 5.TF.36 T, since $(A^T A)^T = A^T (A^T)^T = A^T A$, by Theorem 5.3.9a.

Ch 5.TF.37 F. Verify that matrices $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ are similar.

Ch 5.TF.38 F. Consider $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. The correct formula $\text{im}(B) = \text{im}(BB^T)$ follows from Theorems 5.4.1 and 5.4.2.

Ch 5.TF.39 T. We know that $A^T = A$ and $S^{-1} = S^T$. Now $(S^{-1}AS)^T = S^T A^T (S^{-1})^T = S^{-1}AS$, by Theorem 5.3.9a.

Ch 5.TF.40 T. By Theorem 5.4.2, we have $\ker(A) = \ker(A^T A)$. Replacing A by A^T in this formula, we find that $\ker(A^T) = \ker(AA^T)$. Now $\ker(A) = \ker(A^T A) = \ker(AA^T) = \ker(A^T)$.

Ch 5.TF.41 T. We attempt to write $A = S + Q$, where S is symmetric and Q is skew-symmetric. Then $A^T = S^T + Q^T = S - Q$. Adding the equations $A = S + Q$ and $A^T = S - Q$ together gives $2S = A + A^T$ and $S = \frac{1}{2}(A + A^T)$. Similarly we find $Q = \frac{1}{2}(A - A^T)$. Check that the decomposition $A = S + Q = (\frac{1}{2}(A + A^T)) + (\frac{1}{2}(A - A^T))$ does the job.

Ch 5.TF.42 T. Apply the Cauchy-Schwarz inequality (squared), $(\vec{x} \cdot \vec{y})^2 \leq \|\vec{x}\|^2 \|\vec{y}\|^2$, to $\vec{x} = \begin{bmatrix} x_1 \\ \dots \\ x_n \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 1 \\ \dots \\ 1 \end{bmatrix}$ (all n entries are 1).

Ch 5.TF.43 T. Let $A = \begin{bmatrix} x & y \\ z & t \end{bmatrix}$. We know that $AA^T = A^2$, or $\begin{bmatrix} x^2 + y^2 & xz + yt \\ xz + yt & z^2 + t^2 \end{bmatrix} = \begin{bmatrix} x^2 + yz & xy + yt \\ zx + tz & yz + t^2 \end{bmatrix}$. We need to show that $y = z$. If $y \neq 0$, this follows from the equation $x^2 + y^2 = x^2 + yz$; if $z \neq 0$, it follows from $z^2 + t^2 = yz + t^2$; if both y and z are zero, we are all set.

Ch 5.TF.44 T, since $\vec{x} \cdot (\text{proj}_V \vec{x}) = (\text{proj}_V \vec{x} + (\vec{x} - \text{proj}_V \vec{x})) \cdot \text{proj}_V \vec{x} = \|\text{proj}_V \vec{x}\|^2 \geq 0$. Note that $\vec{x} - \text{proj}_V \vec{x}$ is orthogonal to $\text{proj}_V \vec{x}$, by the definition of a projection.

Ch 5.TF.45 T. Note that $1 = \left\| A \left(\frac{1}{\|\vec{x}\|} \vec{x} \right) \right\| = \left\| \frac{1}{\|\vec{x}\|} A\vec{x} \right\| = \frac{1}{\|\vec{x}\|} \|A\vec{x}\|$ for all nonzero \vec{x} , so that $\|A\vec{x}\| = \|\vec{x}\|$. See Definition 5.3.1.

Ch 5.TF.46 T. If $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ is a symmetric matrix, then $A - xI_2 = \begin{bmatrix} a-x & b \\ b & c-x \end{bmatrix}$. This matrix fails to be invertible if (and only if) $\det(A - xI_2) = (a-x)(c-x) - b^2 = 0$. We use the quadratic formula to find the (real)

solutions $x = \frac{a+c \pm \sqrt{(a+c)^2 - 4ac + 4b^2}}{2} = \frac{a+c \pm \sqrt{(a-c)^2 + 4b^2}}{2}$. Note that the discriminant $(a-c)^2 + 4b^2$ is positive or zero.

Ch 5.TF.47 T; one basis is: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

Ch 5.TF.48 F; A direct computation or a geometrical argument shows that $Q = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, representing a reflection, not a rotation.

Ch 5.TF.49 F; $\dim(\mathbb{R}^{3 \times 3}) = 9$, $\dim(\mathbb{R}^{2 \times 2}) = 4$, so $\dim(\ker(L)) \geq 5$, but the space of all 3×3 skew-symmetric matrices has dimension of 3.

(A basis is $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$.)

Ch 5.TF.50 T; Consider an orthonormal basis \vec{v}_1, \vec{v}_2 of V , and a unit vector \vec{v}_3 perpendicular to V , and form the orthogonal matrix $S = [\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3]$. Now $AS = [\vec{v}_1 \ \vec{v}_2 \ \vec{0}] = S \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Since S is orthogonal, we have

$S^T AS = S^{-1} AS = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, a diagonal matrix.