4.2: Linear Programming
Math 111

1.) A farm co-op has 6000 acres available to plant with corn and soybeans. Each acre of corn requires 9 gallons of fertilizer/herbicide and ¾ hour of labor to harvest. Each acre of soybeans requires 3 gallons of fertilizer/herbicide and 1 hour of labor to harvest. The co-op has available at most 40,500 gallons of fertilizer/herbicide and at most 5250 hours of labor for harvesting. If the profits per acre is $60 for corn and $40 for soybeans, how many acres of each crop should the co-op plant in order to maximize their profit? <u>What is the maximum profit?</u>

STEP 1: Read the problem and don't freak out!!!

STEP 2: Identify your variables.

c = # of acres of corn
s = # of acres of soybean

STEP 3: Note that you are trying to find the maximum profit. Write the profit function from the last sentence. This is also called the *objective* function.

$60c + 40s = P$ (profit).

STEP 4: Find the *constraint* equations. You can use a table to organize your data and come up with these inequalities.

fertilizer:	$9c + 3s$	≤ 40500 gal
acres:	$c + s$	≤ 6000 acres
labor:	$\frac{3}{4}c + 1s$	≤ 5250 hours

STEP 5: Graph the constraints using intercepts. Make sure you label each line. And shade the *feasible* region.

① $9c + 3s \le 40500$

c	s
0	13500
4500	0

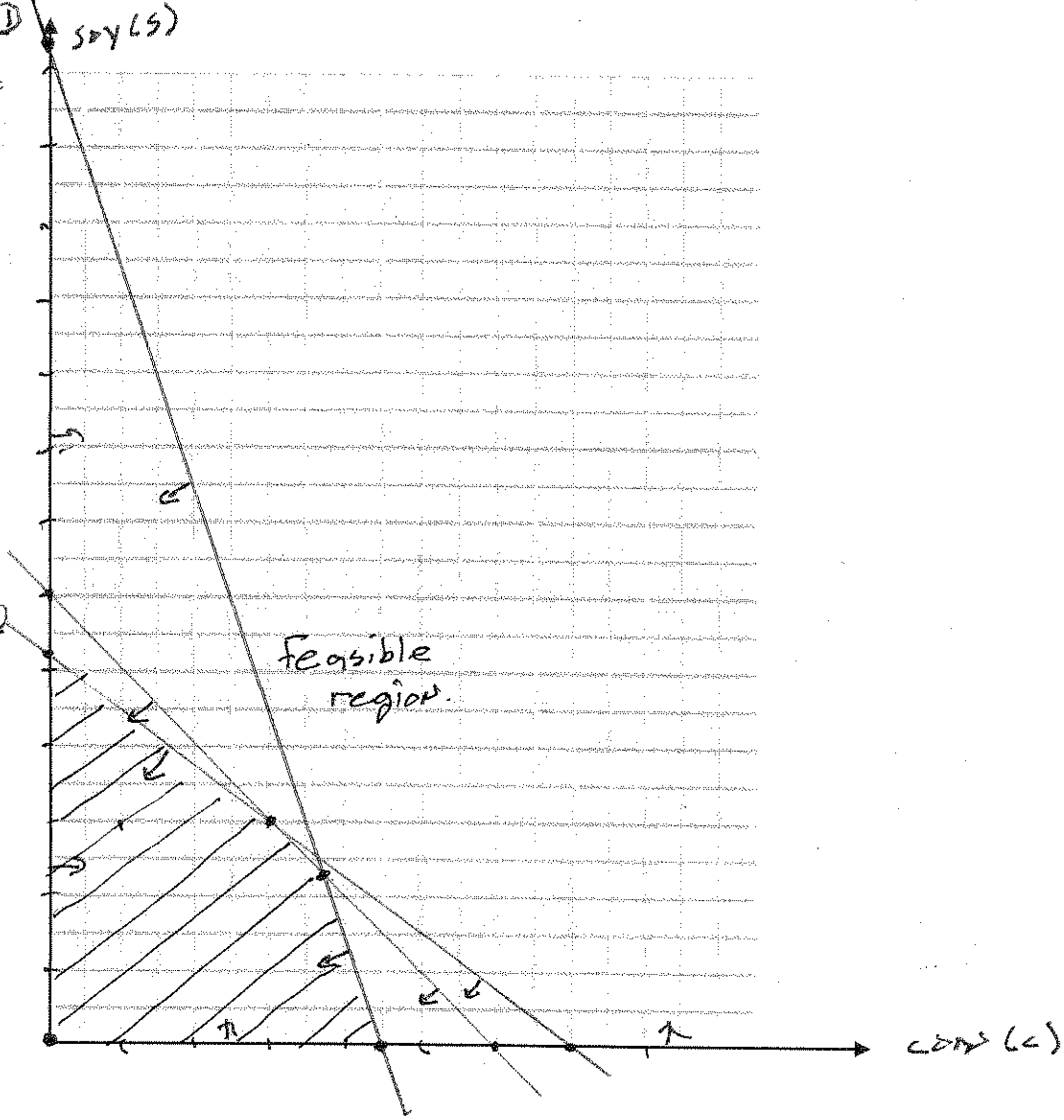
② $c + s \le 6000$

c	s
0	6000
6000	0

③ $\frac{3}{4}c + s \le 5250$

c	s
0	5250
7000	0

$c \ge 0$; $s \ge 0$



STEP 6: Find the corners of the feasible region. Note that they are the points that two constraints meet. So either the intercepts or solve the system using matrices.

(0, 0)

$$\begin{array}{c} ① \\ ② \end{array} \begin{bmatrix} 9 & 3 & 40500 \\ 1 & 1 & 6000 \end{bmatrix} \rightarrow (3750, 2250)$$

(0, 5250)

(4500, 0)

$$\begin{array}{c} ② \\ ③ \end{array} \begin{bmatrix} 1 & 1 & 6000 \\ \frac{3}{4} & 1 & 5250 \end{bmatrix} \rightarrow (3000, 3000)$$

STEP 7: Substitute the corners into profit function and find the profit made on each corner.

$(0,0) \rightarrow P=0$

$(0, 5250) \rightarrow P = 210{,}000$

$(4500, 0) \rightarrow P = 270{,}000$

$\boxed{(3750, 2250) \rightarrow 315{,}000}$

$(3000, 3000) \rightarrow 300{,}000$

MAX PROFIT: $P = 60c + 40s$

STEP 8: Choose the maximum profit.

max profit is \$315,000.

STEP 9: Write your solution in a form of a sentence.

The co-op should plant 3750 acres of corn & 2250 of soy beans to get a max profit of \$315,000.

2.) A company manufactures two types of electric hedge trimmers, one of which is cordless. The cord-type trimmer requires 2 hours to make, and the cordless model requires 6 hours. The company has only 600 work hours to use in manufacturing each day, and the packaging department can package only 200 trimmers per day. If the company sells the cord-type model for \$22.50 and the cordless model for \$67.50, how many of each type should it produce per day to maximize its sales? What is the maximized revenue?

step 2: C = # of corded trimmers

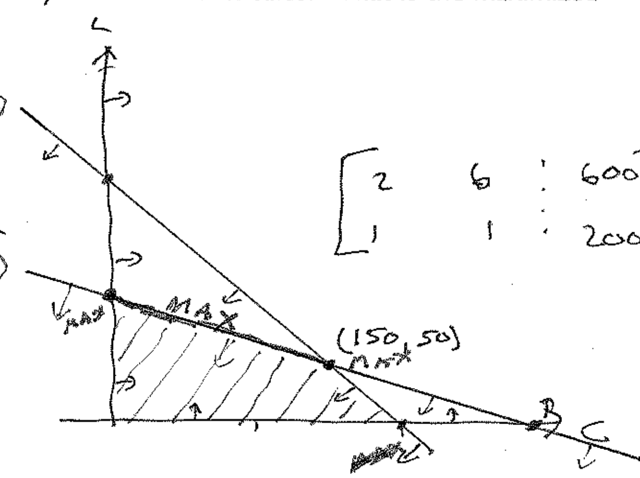
L = # of cordless trimmers

step 3: revenue: $R = 22.5C + 67.5L$

step 4: hours $2C + 6L \le 600$

packing $C + L \le 200$

$C \ge 0$ & $L \ge 0$



$$\left[\begin{array}{cc|c} 2 & 6 & 600 \\ 1 & 1 & 200 \end{array}\right]$$

① $2C + 6L = 600$

C	L
0	100
300	0

② $C + L = 200$

C	L
0	200
200	0

step 6–8:

$(0,0) \rightarrow R = 0$

$(0,100) \rightarrow R = 6750$ MAX

$(200,0) \rightarrow R = 4500$

$(150,50) \rightarrow R = 6750$ MAX

Revenue is maxed @ \$6750 if 100 cordless, or 150 corded and 50 cordless, or any proportional amount is produced.

3.) A candidate wishes to use a combination of radio and television advertisements in her campaign. Research has shown that each 1-minute spot on television reaches 0.09 million people and that each 1-minute spot on radio reaches 0.006 million. The candidate feels she must reach at least 10.8 million people, and she must buy a total of at least 400 minutes of advertisements. How many minutes of each medium should be used to minimize costs if television costs \$400/minute and radio costs \$100/minute?

step 2: variables

T = # of minutes of TV

R = # of " " Radio

step 3: objective fct

cost: $C = 400T + 100R$

step 4: constraints.

\# of people reached: $0.09T + 0.006R \geq 10.8$

\# of total minutes. $T + R \geq 400$

$T \geq 0$; $R \geq 0$

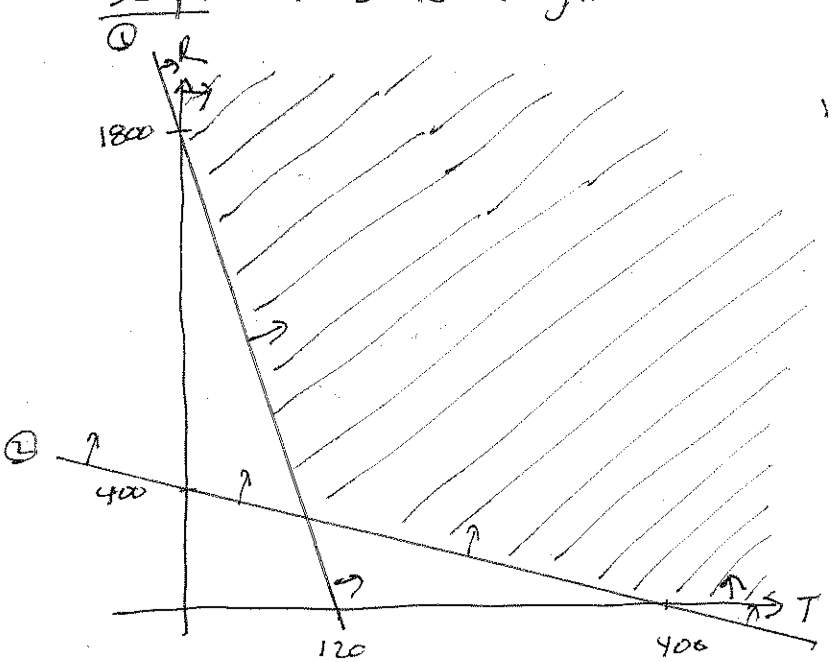
① $0.09T + 0.006R = 10.8$

T	R
0	1800
120	0

② $T + R = 400$

T	R
0	400
400	0

step 5: feasible region



$$\begin{bmatrix} 0.09 & 0.006 & 10.8 \\ 1 & 1 & 400 \end{bmatrix}$$

Step 6-8:

$(0, 1800) \rightarrow C = 180{,}000$

$(400, 0) \rightarrow C = 160{,}000$

$\boxed{(100, 300) \rightarrow C = 70{,}000 \quad \text{min}}$

step 9: we can min cost @ \$70,000 by buying 100 min of TV & 300 min of Radio.

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Minimize $g = 3x + 8y$ subject to $\begin{cases} 4x - 5y \ge 50 \\ -x + 2y \ge 4 \\ x + y \le 80 \end{cases}$

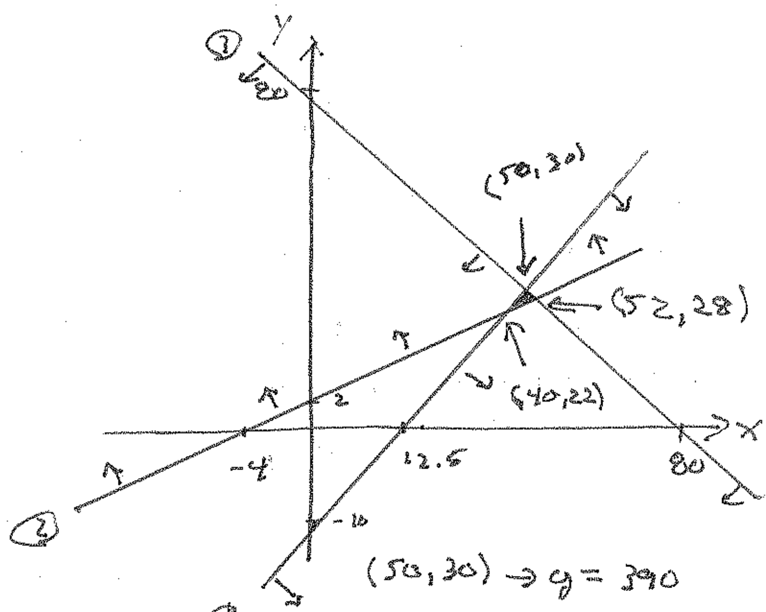
① $4x - 5y = 50$ ② $-x + 2y = 4$

x	y
0	-10
12.5	0

x	y
0	2
-4	0

③ $x + y = 80$

x	y
0	80
80	0



$(50,30) \to g = 390$

$(52,28) \to g = 380$

$\boxed{(40,22) \to g = 296}$

Ikea manufactures two "high-quality" products: rockers and bookshelf units. Its profit is \$30 per rocker and \$42 per bookshelf unit. Next week's production will be constrained by two limited resources, labor and wood. The labor available is expected to be at most 930 hours, and the amount of wood available is expected to be at most 2400 board feet. Each rocker requires 4 labor hours and 8 board feet of wood to produce. Each bookshelf unit requires 3 labor hours and 12 board feet of wood. Find out how many of each type of unit should be produced to maximize the weekly profit.

R = # of rockers
B = # of bookshelves

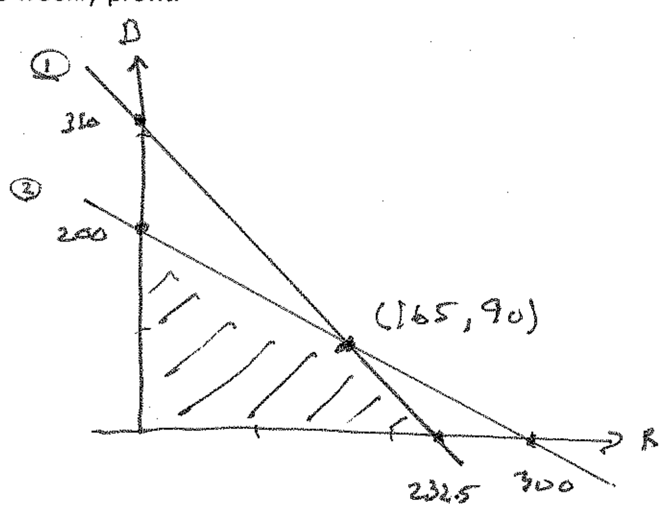
Profit: $P = 30R + 42B$

constraints

labor $4R + 3B \le 930$

~~hours~~ WOOD $8R + 12B \le 2400$

$R \ge 0$; $B \ge 0$.



① $4R + 3B = 930$ ② $8R + 12B = 2400$

R	B
0	310
232.5	0

R	B
0	200
300	0

max profit @ \$8730 when 165 rockers & 90 bookshelves are produced.

$(0,0) \to P = 0$ $(0,200) \to P = 8400$

$(232.5,0) \to P = 6975$ $(165,90) \to P = 8730$

Centex Homes builds two types of homes. The Carolina requires one lot, \$160,000 capital, and 160 worker-days of labor, whereas the Savannah requires one lot, \$240,000 capital, and 160 worker-days of labor. The contractor owns 300 lots and has \$48,000,000 available capital and 43,200 worker-days of labor. The profit on the Carolina is \$40,000 and the profit on the Savannah is \$50,000. Find how many of each type of home should be built to maximize profit. Find the maximum possible profit.

c = # of Carolinas

s = # of Savannahs

Profit: $P = 40c + 50s$ (in \$1000s)

constraints:

lots: $c + s \le 300$

labor: $160c + 160s \le 43{,}200$

\$: $160c + 240s \le 48000$

also $c \ge 0$; $s \ge 0$

① $c + s = 300$

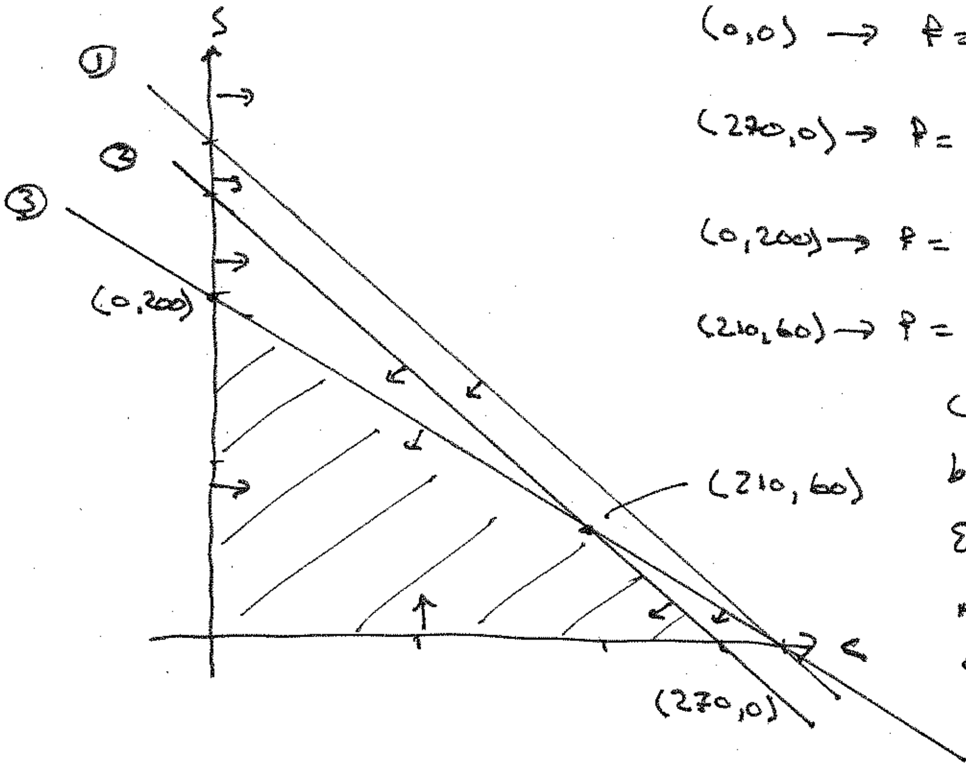
c	s
0	300
300	0

② $160c + 160s = 43200$

c	s
0	270
270	0

③ $160c + 240s = 48000$

c	s
0	200
300	0



$(0,0) \rightarrow P = 0$

$(270,0) \rightarrow P = 10{,}800{,}000$

$(0,200) \rightarrow P = 10{,}000{,}000$

$(210,60) \rightarrow P = 11{,}400{,}000$

Centex maximizes profit by building 210 Carolinas & 60 Savannas @ \$11,400,000. Note: 30 lots are left empty.