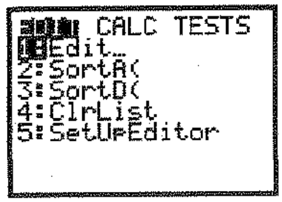
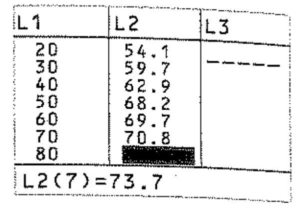
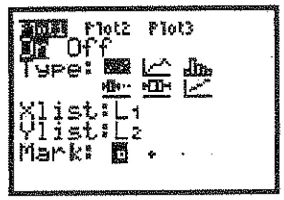
Section 2.5 Modeling

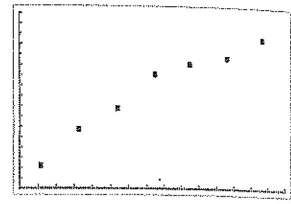
A. To Create a Scatter Plot

1. Press STAT and under EDIT press 1:Edit. This brings you to the screen where you enter data into lists.
2. Enter the x-values (input) in the column headed L1 and the corresponding y-values (output) in the column headed L2.
3. Go to the Y= menu and turn off or clear any functions entered there. To turn off a function, move the cursor over the = sign and press ENTER.
4. Press 2nd STAT PLOT, 1:Plot 1. Highlight ON, and then highlight the first graph type (Scatter Plot), Enter Xlist:L1, Ylist:L2, and pick the point plot mark you want.
5. Choose an appropriate WINDOW for the graph and press GRAPH, or press ZOOM, 9:ZoomStat to plot the data points.



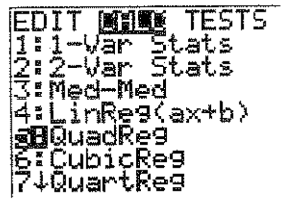


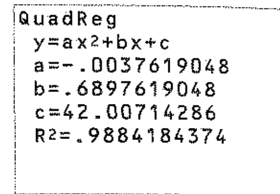


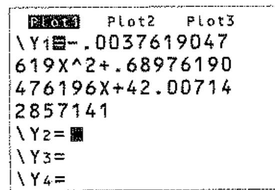


B. To Find an Equation That Models a Set of Data Points

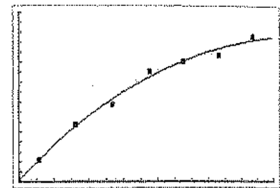
1. Observe the scatter plot to determine what type function would best model the data. Press STAT, move to CALC, and select the function type to be used to model the data.
2. Press the VARS key, move to Y-VARS, and select 1:Function and 1:Y_1. Press ENTER.
   The coefficients of the equation will appear on the screen and the regression equation will appear as Y_1 on the Y = screen.







Pressing ZOOM 9 shows how well the model fits the data.



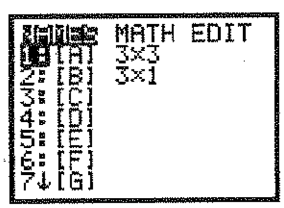
The model is $y = -0.00376x^2 + 0.690x + 42.007$.

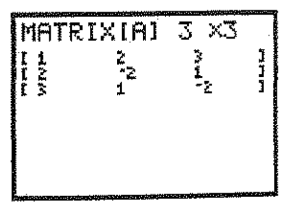
Chapter 3

Section 3.1 Entering Data into Matrices

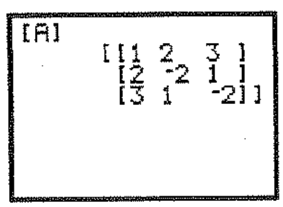
To enter data into matrices, press the MATRIX key. Move the cursor to EDIT. Enter the number of the matrix into which the data is to be entered. Enter the dimensions of the matrix, and enter the value for each entry of the matrix. Press ENTER after each entry.
For example, we enter the matrix below as [A].

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \\ 3 & 1 & -2 \end{bmatrix}$$





1. Enter 3's to set the dimension, and enter the numbers.
2. To perform operations with the matrix or leave the editor, first press 2nd QUIT.
3. To view the matrix, press MATRIX, the number of the matrix, and ENTER.



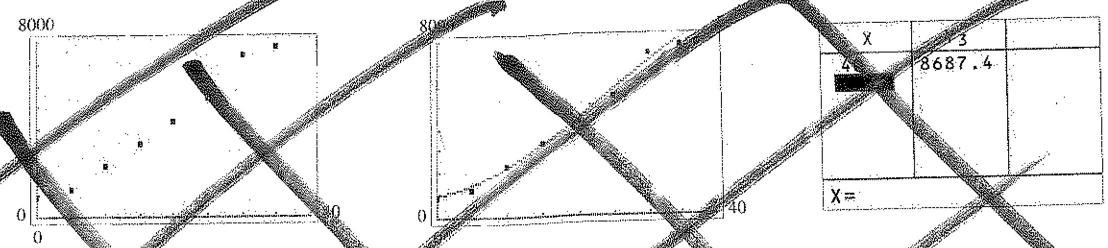


Figure 2.31 (a) (b) (c)

If it is not obvious what model will best fit a given set of data, several models can be developed and compared graphically with the data.

● **EXAMPLE 4** ***Expected Life Span***

The expected life span of people in the United States depends on their year of birth.

Year	Life Span (years)	Year	Life Span (years)	Year	Life Span (years)	Year	Life Span (years)
1920	54.1	1960	69.7	1985	74.7	2005	77.9
1930	59.7	1970	70.8	1990	75.4	2007	77.9
1940	62.9	1975	72.6	1995	75.8		
1950	68.2	1980	73.7	2000	77.0		

Source: National Center for Health Statistics

(a) Create linear and quadratic models that give life span as a function of birth year with $x = 0$ representing 1900 and, by visual inspection, decide which model gives the better fit.
(b) Use both models to estimate the life span of a person born in the year 2000.
(c) Which model's prediction for the lifespan in 2010 seems better?

Solution

(a) The scatter plot for the data is shown in Figure 2.32(a). It appears that a linear function could be used to model the data. The linear equation that is the best fit for the data is

$$y = 0.251x + 52.68$$

The graph in Figure 2.32(b) shows how well the line fits the data points. The quadratic function that is the best fit for the data is

$$y = -0.00213x^2 + 0.530x + 45.2$$

Its graph is shown in Figure 2.32(c).

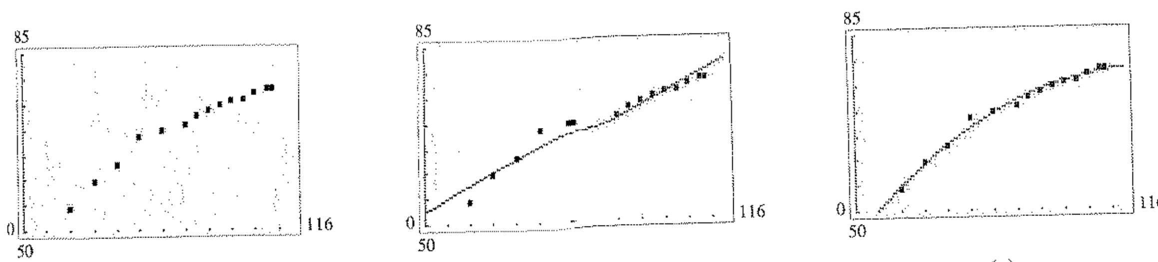


Figure 2.32 (a) (b) (c)

After the type of function that gives the best fit for the data is chosen, a graphing utility or spreadsheet can be used to develop the best-fitting equation for the function chosen. The following steps are used to find an equation that models data.

Modeling Data

1. Enter the data and use a graphing utility or spreadsheet to plot the data points. (This gives a scatter plot.)
2. Visually determine what type of function (including degree, if it is a polynomial function) would have a graph that best fits the data.
3. Use a graphing utility or spreadsheet to determine the equation of the chosen type that gives the best fit for the data.
4. To see how well the equation models the data, graph the equation and the data points on the same set of axes. If the graph of the equation does not fit the data points well, another type of function may model the data better.
5. After the model for a data set has been found, it can be rounded for reporting purposes. However, use the unrounded model in graphing and in calculations, unless otherwise instructed. Numerical answers found using a model should be rounded in a way that agrees with the context of the problem and with no more accuracy than the original output data.

EXAMPLE 3 ***Federal Tax per Capita (Application Preview)***

The table below gives the amount of federal tax paid per capita (per person) for selected years from 1970 to 2005.

(a) Create a scatter plot of these data.
(b) Find a cubic model that fits the data points, with $x = 0$ representing 1970.
(c) Use the model to predict the per capita tax for 2010.

Year	Federal Tax per Capita	Year	Federal Tax per Capita
1970	$955	1990	$4208
1975	1376	1995	5144
1980	2276	2000	7404
1985	3099	2005	7625

Source: Internal Revenue Service

Solution

(a) The scatter plot is shown in Figure 2.31(a). The "turns" in the graph indicate that a cubic function would be a good fit for the data.

(b) The cubic function that is the best fit for the data is

$$y = -0.1384x^3 + 9.660x^2 + 27.17x + 1005$$

The graph of the function and the scatter plot are shown in Figure 2.31(b).

(c) The value of y for $x = 40$ (shown in Figure 2.31(c)) predicts about $8687 as the tax per capita in 2010.