9:00
9:23

Test 1
Dusty Wilson
Math 220

No work = no credit

Name: KEY

The object of pure physics is the unfolding of the laws of the intelligible world; the object of pure mathematics that of unfolding the laws of human intelligence.

James Joseph Sylvester
1814 – 1897 (English mathematician)

Warm-ups (1 pt each): $-2^4 =$ -16 $\quad 1\cdot 3+2\cdot 4=$ 11 $\quad \begin{bmatrix}1 & 2\end{bmatrix}\cdot\begin{bmatrix}3\\4\end{bmatrix}=$ [11]

1.) (1 pts) According to Sylvester (above), what is the difference between pure math and physics? Answer using complete English sentences.

physics is about nature while math is about human intelligence.

2.) (20 pts) Consider the equation $A\cdot x = b$.

a.) What are two approaches you could use to solve this equation?

a. Method 1:

row reduce [A|b]

3/5 for x = rref([A|b])

b. Method 2:

Find A^{-1}. Then $x = A^{-1}b$

left mult. 3/5

b.) With respect to your methods, will these work all of the time or are there conditions on A ~~and b in order~~ required for the method to be successful?

a. Method 1: 3/5

[A|b] must be consistent to have a solution.

no conditions on A.

b. Method 2:

A must be (n×n) & nonsingular/invertible.

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3.) (10 pts) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$.

a.) Solve the equation $A \cdot x = b$ for x.

$$X = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

b.) Set up the augmented matrix you would use to find A^{-1}.

$$[A|I] = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 3 & 3 & 0 & 1 & 0 \\ 1 & 3 & 5 & 0 & 0 & 1 \end{bmatrix}$$

~~a.) Is the matrix A symmetric?~~

~~Yes.~~

4.) (10 pts) Write an augmented matrix in RREF whose general solution in vector form is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 11 \\ 0 \\ 9 \\ 1 \end{bmatrix}$$

$X_1 = 7 + 3x_2 + 11x_4$

X_2 arb.

$X_3 = 5 + 9x_4$

X_4 arb.

2pts – RREF

2pts – (neg)

$$\begin{bmatrix} 1 & -3 & 0 & -11 & 7 \\ 0 & 0 & 1 & -9 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

signs off

25 full diag...

4/10

Arb

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5.) (5 pts) When we solve equations by factoring, we make use of the zero product rule which states that $a \cdot b = 0$ iff $a = 0$ or $b = 0$. This is true for scalars a and b, but is not true for matrices. Give an example of two non-zero matrices whose product is the zero matrix.

example.

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

6.) (10 pts) Show that the vectors $v_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 19 \\ -13 \\ +6 \end{bmatrix}$ are linearly dependent.

$$\text{rref}\left(\begin{bmatrix} 1 & 4 & 19 \\ 3 & -2 & -13 \\ -1 & 3 & +16 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

a square matrix must be row equiv. to I to have L.I. cols.

Also $V_3 = 5V_2 - V_1$

7.) (~~3~~ 9 pts) Prove that if A is an $(m \times n)$ matrix and C is an $(n \times p)$ matrix, then $(A \cdot C)^T = C^T A^T$.

$\square$ proof.

The dim of $(AC)^T$ are $(p \times m)$ which matches $C^T A^T$'s dims.

Now $((A \cdot C)^T)_{ij} = (AC)_{ji}$

$$= \sum_{k=1}^{n} a_{jk} \cdot c_{ki}$$

$$= \sum_{k=1}^{n} c_{ki} \cdot a_{jk}$$

$$= \sum_{k=1}^{n} (C^T)_{ik} \cdot (A^T)_{kj}$$

$= (C^T A^T)_{ij}$

QED ■

+1 notation

+1 dim.

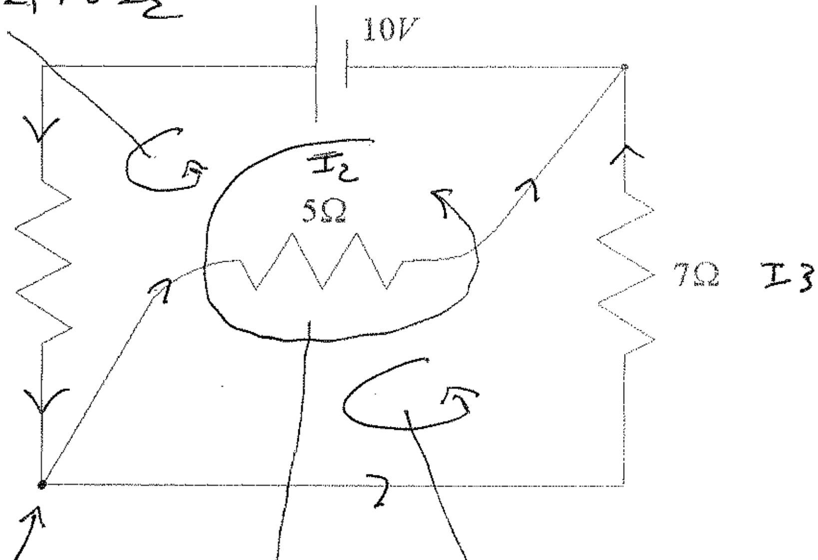
+1 trying.

+2 if clear NTS

15 + X

8.) (~~8~~ 10 pts) Determine the currents thru the three resistors.

$10 = 3I_1 + 5I_2$



$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 3 & 5 & 0 & 10 \\ 3 & 0 & 7 & 10 \\ 0 & -5 & 7 & 0 \end{bmatrix}$$

$I_1 = 120/71$

$I_2 = 70/71$

$I_3 = 50/71$

$I_1 = I_2 + I_3$

$0 = 7I_3 - 5I_2$

$10 = 3I_1 + 7I_3$

9.) (5 pts) Let A be the nonsingular (4×4) matrix $A=[A_1, A_2, A_3, A_4]$ and let $B=[A_1, A_4, A_2, A_3]$. For a given vector b, what is the solution of $A\cdot x = b$ if the solution to $B\cdot x = b$ is: $x=\begin{bmatrix}2\\4\\6\\8\end{bmatrix}$.

$Ax = b$

$x_1A_1 + x_2A_2 + x_3A_3 + x_4A_4 = b$

$\quad 2 \qquad 6 \qquad 8 \qquad 4$

$Bx = b$

$\Rightarrow [A_1, A_4, A_2, A_3]\begin{bmatrix}2\\4\\6\\8\end{bmatrix} = b$

$\Rightarrow 2A_1 + 4A_4 + 6A_2 + 8A_3 = b$

$x = \begin{bmatrix}2\\6\\8\\4\end{bmatrix}$

10.) (10 pts) Find A if A is (2×2) and $(5A)^{-1} = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$.

$$(\underbrace{(5A)^{-1}}_{(\frac{1}{5}A^{-1})^{-1}})^{-1} = 5A = -\frac{1}{2}\begin{bmatrix}4 & -2\\-3 & 1\end{bmatrix}$$

$$\Rightarrow A = -\frac{1}{2(5)}\begin{bmatrix}4 & -2\\-3 & 1\end{bmatrix} = \begin{bmatrix}-2/5 & 1/5\\+3/10 & -1/10\end{bmatrix}$$

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