2.3: Matrix Multiplication

recall: $\underset{n\times m}{A}\ \underset{m\times p}{B} = \begin{bmatrix} — & \vec{a}_1 & — \\ & \vdots & \\ — & \vec{a}_n & — \end{bmatrix}_{n\times m} \begin{bmatrix} | & & | \\ \vec{b}_1 & \cdots & \vec{b}_p \\ | & & | \end{bmatrix}_{m\times p}$

$$= \begin{bmatrix} \vec{a}_1\cdot\vec{b}_1 & \cdots & \vec{a}_1\,\vec{b}_p \\ \vdots & & \vdots \\ \vec{a}_n\,\vec{b}_1 & \cdots & \vec{a}_n\,\vec{b}_p \end{bmatrix}$$

$$= \begin{bmatrix} | & & | \\ A\vec{b}_1 & \cdots & A\vec{b}_p \\ | & & | \end{bmatrix}$$

matrix multiplication is not commutative.

ex1: If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos(\pi/4) & -\sin(\pi/4) \\ \sin(\pi/4) & \cos(\pi/4) \end{bmatrix}$

compute & compare AB and BA. Interpret the answers geometrically using composition diagrams.

Some properties of matrix multiplication/algebra

<u>Thm</u>: multiplying w/ the identity matrix

For $A_{n\times m}$: $A I_m = I_n A = A$

<u>Thm</u>: matrix mult. is assoc.

$$(AB)C = A(BC)$$

<u>Thm</u>: matrix mult. is dist.

(a) $A(C+D) = AC + AD$

(b) $(A+B)C = AC + BC$.

$\square$ proof of (a)

Let $A_{n\times m}$ and $C, D_{m\times p}$.

$$C + D = \begin{bmatrix} | & & | \\ (\vec{c_1 + d_1}) & \cdots & (\vec{c_p + d_p}) \\ | & & | \end{bmatrix}$$

so $$A(C+D) = \begin{bmatrix} | & & | \\ A(\vec{c}_1 + \vec{d}_1) & \cdots & A(\vec{c}_p + \vec{d}_p) \\ | & & | \end{bmatrix}$$

$$= \begin{bmatrix} | & & | \\ (A\vec{c}_1 + A\vec{d}_1) & \cdots & (A\vec{c}_p + A\vec{d}_p) \\ | & & | \end{bmatrix}$$

$$= \begin{bmatrix} | & & | \\ A\vec{c}_1 & \cdots & A\vec{c}_p \\ | & & | \end{bmatrix} + \begin{bmatrix} | & & | \\ A\vec{d}_1 & \cdots & A\vec{d}_p \\ | & & | \end{bmatrix}$$

$$= AC + AD \quad \blacksquare$$

Thm: If $A_{m\times p}$ and $B_{p\times m}$ and k a scalar

$$(kA)B = A(kB) = k(AB).$$

We are going to skip over block-matrices.