4.7
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Optimization

Example 1: A box with an open top is to be constructed from a square piece of cardboard, 3 feet wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

Example 2: A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 feet, find the dimensions of the window so that the greatest possible amount of light is admitted.

Example 3: A woman is at the boat launch of a circular lake with radius 2 miles that has a sidewalk all the way around it. She wants to arrive at the point diametrically opposite the launch (on the other side of the lake) in the shortest possible time. She can walk at the rate of 4 mph and row a boat at 2mph. How should she proceed?

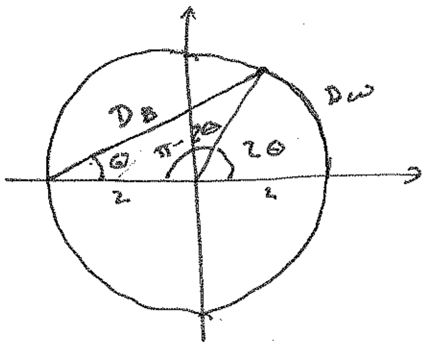
Example 4: Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Example 5: A contractor is engaged to build steps up the slope of a hill that has the shape of the graph $y = \dfrac{x^2(120 - x)}{6400}, 0 \le x \le 80$ with x in meters. What is the maximum vertical rise of a stair if each stair has a horizontal length of one-third meter?

Example 6: An 8-billion-bushel corn crop brings a price of $2.40/bu. A commodity broker uses the rule of thumb: If the crop is reduced by x percent, then the price increases by $10x$ cents. Which crop size results in maximum revenue and what is the price per bushel?

Example 7: A billboard of height b is mounted on the side of a building with its bottom edge at a distance h from the street below. At what distance x should an observer stand from the wall to maximize the angle of observation?

Example 3: A woman is at the boat launch of a circular lake with radius 2 miles that has a sidewalk all the way around it. She wants to arrive at the point diametrically opposite the launch (on the other side of the lake) in the shortest possible time. She can walk at the rate of 4 mph and row a boat at 2mph. How should she proceed?



$D_w = 2 \cdot 2\theta$

$D_B = \sqrt{8 - 8\cos(\pi - 2\theta)}$

$= 2\sqrt{2 - 2\cos(\pi - 2\theta)}$

and $0 \le \theta \le \frac{\pi}{2}$

So $T(\theta) = \frac{D_w}{4} + \frac{D_B}{2}$

$= \theta + \sqrt{2 - 2\cos(\pi - 2\theta)}$

$\Rightarrow T'(\theta) = 1 + \frac{+2\sin(\pi - 2\theta)(-2)}{2\sqrt{2 - 2\cos(\pi - 2\theta)}}$

$= 1 - \frac{2\sin(\pi - 2\theta)}{\sqrt{2 - 2\cos(\pi - 2\theta)}}$

and solve $T'(\theta) = 0$

$\frac{1}{2} = \frac{\sin(\pi - 2\theta)}{\sqrt{2 - 2\cos(\pi - 2\theta)}}$

$\Rightarrow \frac{1}{4} = \frac{\sin^2(\pi - 2\theta)}{2 - 2\cos(\pi - 2\theta)}$

$\Rightarrow \frac{1}{4}(2 - 2\cos(\pi - 2\theta)) = 1 - \cos^2(\pi - 2\theta)$

$\Rightarrow \cos^2(\pi - 2\theta) - \frac{1}{2}\cos(\pi - 2\theta) - \frac{1}{2} = 0$

$\Rightarrow 2c^2 - c - 1 = 0$ where $c = \cos(\pi - 2\theta)$

$\Rightarrow (2c + 1)(c - 1) = 0$

$\Rightarrow c = -\frac{1}{2}$ or $c = 1$

So $\cos(\pi - 2\theta) = 1$

$\Rightarrow \pi - 2\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

or

$\pi - 2\theta = 2\pi \Rightarrow \theta = -\frac{\pi}{2}$ (outside domain)

or $\cos(\pi - 2\theta) = -\frac{1}{2}$

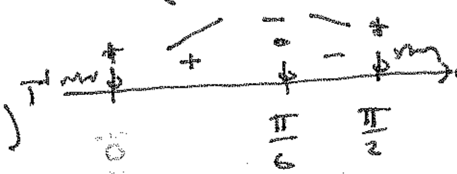
$\Rightarrow \pi - 2\theta = \frac{2\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$

or

$\pi - 2\theta = \frac{4\pi}{3} \Rightarrow \theta = -\frac{\pi}{6}$ (outside domain)

- But T' is undefined when $\cos(\pi - 2\theta) = 1$ or $\theta = \pm\frac{\pi}{2}$
- Did we pick up other extraneous solutions?

$T'(\frac{\pi}{6}) = 0$ & $T'(-\frac{\pi}{6}) = 2$ (extra soln)



There is a max time when $\theta = \frac{\pi}{6}$

hmmm.

check endpoints.

$T(0) = 2$ hrs; $T(\frac{\pi}{2}) \approx 1.57$

She should walk the whole way.

Example 4: Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$