4.3: Derivatives & Graphs

Concepts.

increasing, decreasing
concavity.
points of inflection
2nd deriv. test.

ex1: Find the following for $f(x) = 4x^3 + 3x^2 - 6x + 1$

(a) intervals where f is increasing/decreasing

(b) local max/mins

(c) intervals of concavity & inflection pts.

ex2: Sketch f if:

(a) $f'(1) = f'(-1) = 0$

(b) $f'(x) < 0$ if $|x| < 1$

(c) $f'(x) > 0$ if $1 < |x| < 2$

(d) $f'(x) = -1$ if $|x| > 2$

(e) $f''(x) < 0$ if $-2 < x < 0$

(f) inflection point @ $(0, 1)$.

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2/2

sketch the following

Ex 3: $h(x) = 5x^3 - 3x^5 = x^3(5 - 3x^2)$

$= x^3(\sqrt{5} - \sqrt{3}x)(\sqrt{5} + \sqrt{3}x)$

$\Rightarrow h'(x) = 15x^2 - 15x^4$

$= 15x^2(1 - x^2)$

$= 15x^2(1 - x)(1 + x)$

$\Rightarrow h''(x) = 30x - 60x^3$

$= 30x(1 - 2x^2)$

$= 30x(1 - \sqrt{2}x)(1 + 2\sqrt{x})$

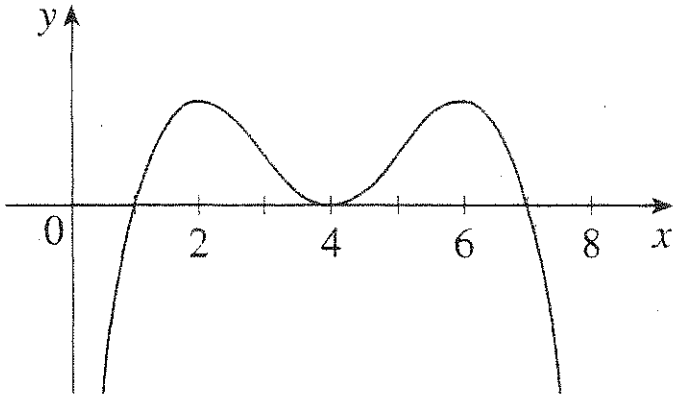
Ex 4: $f(x) = x - \frac{1}{6}x^2 - \frac{2}{3}\ln x$

In each part state the x-coordinates of the inflection points of f. Give reasons for your answers.

(a) The curve is the graph of f.

(b) The curve is the graph of f'.

(c) The curve is the graph of f''.



The graph of the derivative f' of a continuous function f is shown.

(a) On what intervals is f increasing? Decreasing?

(b) At what values of x does f have a local maximum? Local minimum?

(c) On what intervals is f concave upward? Concave downward?

(d) State the x-coordinate(s) of the point(s) of inflection.

(e) Assuming that $f(0) = 0$, sketch a graph of f.