3.5: Implicit Differentiation

Q: How do we find $\frac{dy}{dx}$ if we do not have y as a fct of x?

ex1: $y = \sqrt{b^2 - \frac{b^2}{a^2}x}$ vs. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}\left(\sqrt{b^2 - \frac{b^2}{a^2}x^2}\right)$ vs $\frac{d}{dx}\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = \frac{d}{dx}(1)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(b^2\left(1 - \frac{x^2}{a^2}\right)\right)^{-1/2} \cdot \frac{(-2x)}{b^2/a^2}$ vs $\frac{2x}{a^2} + \frac{2y}{b^2}\cdot\frac{dy}{dx} = 0$

$= \frac{-x \cdot b^2}{a^2\sqrt{b^2\left(1 - \frac{x^2}{a^2}\right)}}$ vs. $\frac{dy}{dx} = -\frac{2x}{a^2} \div \frac{2y}{b^2}$

$= \frac{-x}{\frac{a^2}{b^2}\sqrt{b^2\left(1 - \frac{x^2}{a^2}\right)}}$ $= \frac{-x}{\frac{a^2}{b^2}y}$

$= \frac{-x}{\frac{a^2}{b^2}\cdot \pm\sqrt{b^2\left(1 - \frac{x^2}{a^2}\right)}}$

ex2: Find $\frac{dy}{dx}$ of $2x^3 + x^2y - xy^3 = 2$

ex3: Find the eqt. of the tangent to the hyperbola $x^2 + 2xy - y^2 + x = 2$ @ $(1,2)$

ex4: $\frac{d}{dx}\cos^{-1}(x)$ and $\frac{d}{dx}\arctan(x)$.