Limit law handout

ex1: $$\lim_{x\to 1}\sqrt{\frac{x^2+x}{3-x}} = \sqrt{\lim_{x\to 1}\frac{x^2+x}{3-x}} \quad \text{rule 11}$$

$$= \sqrt{\frac{\lim_{x\to 1}(x^2+x)}{\lim_{x\to 1}(3-x)}} \quad \text{rule 5}$$

$$= \sqrt{\frac{\lim_{x\to 1}x^2+\lim_{x\to 1}x}{\lim_{x\to 1}3-\lim_{x\to 1}x}} \quad \text{rules 1 \& 2}$$

$$= \sqrt{\frac{1+1}{3-1}} \quad \text{rules 7 \& 9}$$

$$= 1.$$

(Graph it to verify).

Direct Substitution Property: If f is a poly or rat. fct & a is in the domain of f then $\lim_{x\to a} f(x) = f(a)$.

(check ex1 again).

ex2: $$\lim_{x\to 3}\frac{x-3}{x^2-x-6}$$

If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$ provided the limits exist.

Remember, limits don't care about individual points!

ex 3: $\lim_{h \to 0} \frac{(1+h)^4 - 1}{h}$

use Pascal's Triangle if time permits.

ex 4: $\lim_{x \to -4} \frac{\sqrt{x^2+9} - 5}{x+4} = \lim_{x \to -4} \frac{\sqrt{x^2+9} - 5}{x+4} \cdot \frac{\sqrt{x^2+9} + 5}{\sqrt{x^2+9} + 5}$

$$= \lim_{x \to -4} \frac{x^2+9-25}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x \to -4} \frac{x-4}{\sqrt{x^2+9}+5}$$

$$= \frac{-8}{10}$$

ex 5: $\lim_{t \to 0} H(t)$ where $H(t)$ is the Heaviside

for $H(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

Thm: If $f(x) \leq g(x)$ when x is near a (except possibly @ $x = a$) and the limits exist, then

$$\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x).$$

<u>The Squeeze thm:</u> If $f(x) \le g(x) \le h(x)$ when x is near a (except possibly @ $x=a$) and

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L \quad \text{then} \quad \lim_{x\to a} g(x) = L$$

It's like corralling a child!

<u>ex6:</u> $\displaystyle\lim_{x\to 0^+} \sqrt{x}\, e^{\sin\left(\frac{\pi}{x}\right)}$. (use the squeeze thm).