Test 2 Key.

$$\sum_{n=1}^{\infty}\left(\frac{\ln n}{n}\right)^{n}$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}{1\cdot 4\cdot 7\cdot\ldots\cdot(3n-2)} \quad \text{or} \quad \sum_{n\geq 1}^{\infty}\frac{(-1)^n}{n(1+(\ln n)^2)}$$

$$\sum_{n=1}^{\infty}\frac{(n+2)!}{3^n(n!)^2}$$

$$\sum_{n=3}^{\infty}\frac{1}{(2n-3)(2n-1)}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n(n^2+1)}{2n^2+n-1}$$

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n(n+1)(n+2)}}$$

Write $2.\overline{71}$ as the ratio of two integers

Does the sequence $\left\{\left(1+\frac{7}{n}\right)^n\right\}$ converge or diverge. If it is convergent, find the limit.

$$\sum_{n=1}^{\infty} \left(\frac{\ln n}{n} \right)^n$$

Root test.

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{\ln n}{n}\right)^n} = \lim_{n\to\infty} \frac{\ln n}{n}$$

$$\overset{H}{=} \lim_{n\to\infty} \frac{1}{n}$$

$$= 0 < 1$$

Hence the series converges by the root test.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (n^2+1)}{2n^2+n-1}$$

Test for Divergence.

$$\lim_{n\to\infty} \frac{(-1)^n (n^2+1)}{2n^2+n-1} = \lim_{n\to\infty} \frac{(-1)^n \left(1+\frac{1}{n^2}\right)}{2+\frac{1}{n}-\frac{1}{n^2}}$$

$$\neq 0$$

so it diverges by the test for divergence.

5/10 if they did the test for divergence unaware.

Does $\left\{\left(1+\frac{7}{n}\right)^n\right\}$ converge?

$$\lim_{n\to\infty}\left(1+\frac{7}{n}\right)^n = e^{\lim_{n\to\infty} n\ln\left(1+\frac{7}{n}\right)}$$

$$= e^{\lim_{n\to\infty} \frac{\ln\left(1+\frac{7}{n}\right)}{1/n}}$$

$$= e^{\lim_{n\to\infty} \frac{\frac{1}{1+\frac{7}{n}}\cdot -\frac{7}{n^2}}{-\frac{1}{n^2}}}$$

$$= e^{\lim_{n\to\infty} \frac{7}{1+\frac{7}{n}}}$$

$$= e^{7}.$$

Yes, it converges.

$$\sum_{n=3}^{\infty} \frac{1}{\sqrt{n(n\mp 1)(n\mp 2)}}$$

Limit comparison test.

$$\lim_{n\to\infty} \frac{\frac{1}{n^{3/2}}}{\frac{1}{\sqrt{n(n-1)(n-2)}}} = \lim_{n\to\infty} \sqrt{\frac{n(n-1)(n-2)}{n^3}} = 1$$

so it converges since $\sum_{n=3}^{\infty} \frac{1}{n^{3/2}}$ is a convergent p-series.

Write $2.\overline{71}$ as the ratio of integers.

$$2.\overline{71} = 2 + 0.71 + 0.0071 + 0.000071 + \ldots$$

$$= 2 + 0.71 \sum_{n=0}^{\infty} \left(\frac{1}{100}\right)^n$$

$$= 2 + \frac{71}{100} \cdot \frac{1}{1 - \frac{1}{100}}$$

$$= 2 + \frac{71}{100} \cdot \frac{100}{99}$$

$$\sum_{n=1}^{\infty} 2 + \frac{71}{100^n}$$

6/10

$$= 2 + \frac{71}{99}$$

$$= \frac{198 + 71}{99}$$

$$= \frac{269}{99}$$

$$\sum_{n=3}^{\infty} \frac{1}{(2n-3)(2n-1)}$$

comparison test

$$\sum_{n=3}^{\infty} \frac{1}{4n^2-8n+3} < \frac{1}{4}\sum_{n=3}^{\infty} \frac{1}{n^2}$$

and so converges by comparison with the convergent p-series.

$$\sum_{n=3}^{\infty} \frac{1}{(2n-3)(2n-1)} = \frac{1}{2}\sum_{n=3}^{\infty}\left(\frac{1}{2n-3} - \frac{1}{2n-1}\right)$$

$$= \frac{1}{2}\left(\frac{1}{3} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} - \cancel{\frac{1}{9}} + \ldots\right.$$

$$= 1/6$$

$$\frac{1}{(2n-3)(2n-1)} = \frac{A}{2n-3} + \frac{B}{2n-1}$$

$$\Rightarrow \quad 1 = 2An - A + 2Bn - 3B$$

$$\Rightarrow \quad 2A + 2B = 0$$

$$A + 3B = -1$$

$$\Rightarrow \quad -B + 3B = -1$$

$$\Rightarrow \quad B = -1/2$$

$$A = 1/2$$

$$\sum_{n=1}^{\infty} \frac{(n+1)!}{3^n (n!)^2} = \sum_{n=1}^{\infty} \frac{(n+1)\cancel{n!}}{3^n \cdot (n!)^{\cancel{2}}}$$

ratio test.

$$\lim_{n\to\infty} \frac{n+2}{3^{n+1}(n+1)!} \cdot \frac{3^n \cdot n!}{n+1} = \lim_{n\to\infty} \frac{n+2}{3(n+1)^2} = 0$$

Hence it converges abs. by the ratio test.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n(1+\ln^2 n)}$$

Alt. series test.

$$\lim_{n\to\infty} \frac{1}{n(1+\ln^2 n)} = 0$$

so it converges conditionally by the A.S.T.

Does it converge absolutely?

$$\int_1^{\infty} \frac{dx}{x(1+\ln^2 x)} = \int_0^{\infty} \frac{du}{1+u^2} = \lim_{t\to\infty} \Big[\arctan u\Big]_0^{t}$$

Let $u = \ln x$

$du = \frac{dx}{x}$

$$= \lim_{t\to\infty} \arctan t - \underbrace{\arctan 0}_{0}$$

$$= \frac{\pi}{2}$$

Thus it converges absolutely by the integral test.

$\frac{5}{10}$ for conditionally convergent by A.S.T.