

### 13.4: Motion in Space: Velocity and Acceleration

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If  $\vec{r}(t)$  gives the position of a particle moving thru space, then (as expected)  $\vec{v}(t) = \vec{r}'(t)$  and  $\vec{a}(t) = \vec{r}''(t)$ .

Ex 1: Find and sketch  $\vec{r}, \vec{v}, \vec{a}$  at  $t = \pi/6$  if

$$\vec{r}(t) = \langle \sin(t), 2 \cos(t) \rangle$$

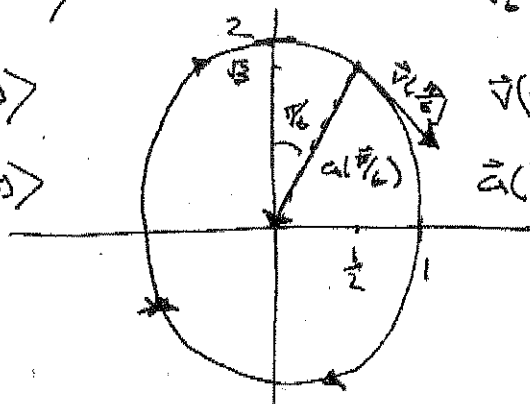
$$\vec{r}(\pi/6) = \langle \frac{1}{2}, \sqrt{3} \rangle$$

$$\vec{v}(t) = \langle \cos(t), -2 \sin(t) \rangle$$

$$\vec{v}(\pi/6) = \langle \frac{\sqrt{3}}{2}, -1 \rangle$$

$$\vec{a}(t) = \langle -\sin(t), -2 \cos(t) \rangle$$

$$\vec{a}(\pi/6) = \langle -\frac{1}{2}, -\sqrt{3} \rangle$$



Ex 2: Find  $\vec{v}(t)$  and  $\vec{r}(t)$  if  $\vec{a}(t) = \langle 0, 0, 1 \rangle$  and  $\vec{v}(0) = \langle 1, -1, 0 \rangle$  and  $\vec{r}(0) = \vec{0}$ .

### Tangential and Normal components of Acceleration

$\vec{v}$  gives velocity and we use  $v = |\vec{v}|$  to represent speed.

$$\text{Now, recall } \vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \frac{\vec{v}}{v}$$

$$\Rightarrow \vec{v} = v \vec{T} \Rightarrow \vec{v}' = \vec{a} = v' \vec{T} + v \vec{T}' *$$

$$\text{also recall } k = \frac{|\vec{T}'|}{|\vec{r}'|} = \frac{|\vec{T}'|}{v} \Rightarrow |\vec{T}'| = k v$$

but we defined the unit normal as  $\vec{N} = \frac{\vec{T}'}{|\vec{T}'|}$

$$\Rightarrow \vec{T}' = \vec{N} |\vec{T}'| = k v \vec{N}$$

\* Hence  $\vec{a} = v' \vec{T} + k v^2 \vec{N}$

Now we can write

$$\vec{a} = a_T \vec{T} + a_N \vec{N} \quad \text{where } a_T = v' \text{ and } a_N = kv^2$$

Find  $a_T$  and  $a_N$  in terms of  $\vec{r}$ .

$$\begin{aligned} a_T: \quad \vec{v} \cdot \vec{a} &= vT \cdot (v' \vec{T} + kv^2 \vec{N}) \\ &= vv' \vec{T} \cdot \vec{T} + kv^3 \vec{T} \cdot \vec{N} \quad (\text{since } \vec{T} \cdot \vec{T} = 1 \\ & \quad \vec{T} \cdot \vec{N} = 0) \\ &= vv' \end{aligned}$$

$$\Rightarrow a_T = v' = \frac{\vec{v} \cdot \vec{a}}{v} = \frac{\vec{r}'(t) \cdot \vec{r}''(t)}{|\vec{r}'(t)|}$$

$$a_N: \quad kv^2 = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3} = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|}$$

Ex 2: A particle moves along the twisted cubic  $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ . Find the tangential and normal components of acceleration.