13.2: Derivatives and Integrals of Vector Functions

If $\vec{r}(t)$ is a vector valued fct, then

$$\vec{r}\,'(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h}$$ provided the limit exists.

Animation of $\vec{r}\,'(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h}$

__Thm:__ If $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ where x, y, z are differentiable fcts, then $\vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle$

__Ex1:__ a) Sketch $\vec{r}(t) = \langle 1+t, \sqrt{t} \rangle$. b) Find $\vec{r}\,'(1)$.
c) Find the unit tangent vector $\vec{T}(1)$ when $t=1$.

A curve is smooth if $\vec{r}\,'(t) \neq 0$ (except possibly at endpoints).

__Ex 2:__ Is $r(t) = \langle \cancel{\frac{t^3}{3} - t}\ 4\cos^3 t, \cancel{\frac{t^2}{3} - t}\ 4\sin^3 t \rangle$ smooth?

__Thm:__ Suppose $\vec{u}$ & $\vec{v}$ are vector valued fcts and f is real valued.

a) $\frac{d}{dt}[c\vec{u}(t) \pm \vec{v}(t)] = c\vec{u}\,'(t) \pm \vec{v}\,'(t)$

b) $\frac{d}{dt}[f(t)\vec{u}(t)] = f'(t)\vec{u}(t) + f(t)\vec{u}\,'(t)$

c) $\frac{d}{dt}[\vec{u}(t) \cdot \vec{v}(t)]$

d) $\frac{d}{dt}[\vec{u}(t) \times \vec{v}(t)]$

e) $\frac{d}{dt}\vec{u}(f(t)) = f'(t)\vec{u}\,'(f(t))$ (chain rule).

Ex3: At what point do $r_1(t) = \langle t, 1-t, 3+t^2 \rangle$ and $r_2(s) = \langle 3-s, s-2, s^2 \rangle$ intersect? Find the angle of intersection.

$$\int_a^b \vec{r}(t)\,dt = \left\langle \int_a^b x(t)\,dt, \int_a^b y(t)\,dt, \int_a^b z(t)\,dt \right\rangle$$

Ex4: $\int \langle e^t, 2t, \ln t \rangle \, dt$.

time permiting, carry on to 13.3.