12.8 : Power Series

A power series is a fct of the form $\sum_{n=0}^{\infty} c_n x^n$ whose domain is the set of all x's for which the series converges.

NOTE: c_n's are coefficients, x is the variable.

Ex1: $\sum_{n=0}^{\infty} c \cdot x^n = c + cx + cx^2 + \dots$

This is a geometric series which converges when $|x| < 1$, and $\sum_{n=0}^{\infty} cx^n = \frac{c}{1-x}$, $|x| < 1$.

Generally: the power series centered at $x = a$ is $\sum_{n=0}^{\infty} c_n (x-a)^n$.

NOTE: When $x = a$, we say $(x-a)^0 = 1$.

Ex 2: When does $\sum_{n=0}^{\infty} n!\, x^n$ converge?

Using the ratio test, we have $\lim_{n\to\infty} \frac{(n+1)!\, x^{n+1}}{n!\, x^n}$

$\hookrightarrow = \lim_{n\to\infty} (n+1)x$. This limit has magnitude less than 1 iff $x = 0$. Otherwise the limit diverges. So the series converges iff $x = 0$.

<u>Ex 3:</u> For what values of x does $\sum_{n=1}^{\infty} \frac{x^n}{n 3^n}$ converge?

* use the ratio test .... $|x| < 3$.
* check endpoints: $x = 3$ divergent
  $x = -3$ convergent.

Since the series converges on $[-3, 3)$, we say the <u>interval of convergence</u> is $[-3, 3)$

<u>Review:</u> What is the interval of convergence in Ex 1 and Ex 2?

<u>Ex 4:</u> Find the interval of convergence of the series $\sum_{n=0} \frac{x^n}{n!}$

<u>Theorem:</u> For a given power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ there are only three possibilities.

i) converges only at $x = a$.

ii) converges for all x.

iii) $\exists R > 0$ s.t. the series converges when $|x-a| < R$ and diverges when $|x-a| > R$. We call R the <u>radius of convergence</u>.

<u>NOTE</u>: Radius of Convergence vs. Interval of convergence

<u>Ex 5:</u> Find the ROC and I.O.C. of $\sum_{n=2}^{\infty} \frac{x^n}{\ln(n)}$

<u>Ex 6:</u> Find the ROC and I.O.C. of $\sum_{n=0}^{\infty} \sqrt{n} (x-1)^n$

Show Cungus example of a drum membrane.